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Summary Report of the

INDIANA CONFERENCE

ON

NUCLEAR SPECTROSCOPY

AND

THE SHELL MODEL

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PREFACE

Plans for the conference were initiated and carried out by A. C. G. Mitchell, Head of the Indiana University Physics Department. More than 100 conferees from 20 laboratories attended.

D. R. Hamilton, Martin Deutsch, Gregory Breit and Eugene Greuling served as chairmen of the four sessions. Twenty-five lectures were given, introductory to informal discussions. The lecturer's names are given in the body of the r eport, underlined when presented in connection with the topics each discussed.

This summary was prepared by E. J. Konopinski, who must be held responsible for any misinterpretations or inaccuracies which may have crept in. The inaccuracies w ere held to a minimum by the able assistance, in taking notes, of H. Mahmoud, D. Moffat, V. Rasmussen, A. Smith and R. G. Wilkinson.

INDIANA CONFERENCE ON NUCLEAR SPECTROSCOPY AND THE SHELL MODEL

An objective for the conference was aptly expressed by Goldhaber: to distinguish between "fact and fiction", in the interpretation of the vast accumulation of data on the behavior of transforming nuclei and their radiations.

The "facts" of nuclear spectroscopy grow not only in quantity but in variety and refinement. The study of γ-rays now includes, besides energy, intensity and lifetime measurements: 1) $\gamma\gamma$ coincidences, 2) directional correlation of successive γ-rays, 3) correlation of polarization and direction, 4) correlation between γ-rays and internal conversion electrons, 5) internal conversion intensities, 6) K/L conversion ratios, 7) $L_I/L_{II,III}$ conversion ratios, 8) coincidences of γ-rays with protons in reactions, with α-particles and β-rays, 9) correlations of γ with β or α emission directions. The β-radiation studies discussed also included: 10) lifetime-energy relations, 11) spectrum shape measurements, 12) β-nuclear recoil direction correlations, 12) Auger electron to positron ratios. Data on nuclear bombardments which were specifically discussed included: 13) resonance widths in scattering, 14) angular distributions in deuteron stripping, 15) directional correlation of fission with the initiating neutrons. Finally, static properties of nuclei, such as magnetic and quadrupole moments, were introduced as evidence.

Some "fiction" inevitably became mixed with fact when, for example, interpretations were restricted to conformity with the primitive version of the nuclear shell model. It is laudable to avoid "ad hoc" assumptions, but, by now, exceptions which call for further assumptions have grown into well-defined classes of cases. A gradually more complex picture of nuclear structure is emerging, but also a more detailed and satisfying one.

In the primitive shell model, all the paired like nucleons are treated as forming the inert core. For an odd A nucleus, this leaves one odd neutron or proton to be assigned to a specific orbital state: $\ell_{j = \ell \pm \frac{1}{2}}$. Such single particle states still seem sufficient to characterize the odd A nuclei in many connections. Examples actually discussed are listed:

(a) The mirror nuclei which consist of (filled orbitals) $\pm$ (one nucleon). The favored β-transitions between such isobars have comparative half-lives quite closely consistent with single particle states. (See below, THE MIRROR NUCLEI.)

(b) The ratios between β-transitions with different degrees of forbiddenness. The order of magnitudes of these ratios seems to be roughly explainable by treating the transitions as between single-nucleon states. (See Unfavored Factor for Heavier Nuclei.)

(c) The "Class I, odd A" allowed β-transitions, as defined by Nordheim. These are cases in which only the last odd nucleon must change its orbital during the transition. They tend to have shorter half-lives than "Class II, odd A" nuclei. The latter are conceived to have differently filled cores, as implied by the odd nucleon assignments. (See Classification of Odd-A Allowed Transitions.)

(d) The low excited levels of odd A nuclei. The excitation energies assigned the same ℓ_j show a smooth variation as neutron pairs are added to the core. (See Odd A Nuclei.)

(e) The resonances in the elastic scattering of protons by even-even nuclei. Some of the resonances have energy-widths characteristic of a single proton state. Interspersed with such are also distinctly narrower resonances. In the latter states, the energy is presumably shared by many nucleons and the lifetime consequently protracted. (See Resonance Scattering of Protons.)

(f) The deuteron stripping (d, p) reactions. In these, the neutron is captured into a nuclear orbital, the character of which determines the observed angular distribution of the outgoing protons. In many cases (usually for even A targets) a neutron of a definite orbital angular momentum ℓ is found to be captured. Captures by odd A target nuclei were discussed, and small admixtures of a second ℓ-value found — attesting to small deviations from the single orbital characterization. (See Stripping Reactions.)

For even A nuclei, more than one nucleon must be considered as determining the state. Even-even nuclei all appear to have spin I = 0 ground states, as expected for cores. Odd-odd nuclei have two unlike odd nucleons: $\ell_j ; \ell'_{j'}$. The question arises as to whether the resultant state character is formed from the individual states of given j and j' (jj-coupling) or whether total spin S and total angular moment L are conserved, yielding a $^{2S+1}L_I$ character (LS-coupling). Strong individual spin-orbit coupling seems necessary to obtain the so-called "magic numbers". This implies that L and S cannot be good quantum numbers and so lends support to the jj-coupling scheme. It may still be that, for some states of light nuclei, the LS coupling provides the better approximation (see ft-Values and Magnetic Moments, and The jj-Coupling Results). The problem of the resultant states of odd-odd nuclei was discussed in the following contexts:

(g) The half-lives of favored β-transitions were compared as between states calculated with jj-coupling, and also as connecting LS-coupled states. Both types of resultant states work about equally well; neither is completely satisfactory. (See The jj-Coupling Results.)

(h) Nordheim had found empirically that for "Class I, even A" nuclei, defined as having $j = \ell \pm \frac{1}{2}$ and $j' = \ell' \mp \frac{1}{2}$, the total spin is usually $I = |j - j'|$. "Class II, even A" nuclei, for which $j = \ell \pm \frac{1}{2}$ and $j' = \ell' \pm \frac{1}{2}$, have $I \rightarrow j + j'$ more nearly. Supporting this are the allowed β-transitions between Class I odd-odd nuclei and the I = 0 ground states of their even-even isobars. In contrast the allowed Class II transitions almost always go to excited states of the even-even isobars, apparently because the ground states are forbidden to them. (See Classification of Even-A Allowed Transitions.)

The obvious next step is to consider states formed from configurations of part or all the nucleons not in filled orbitals. This seems necessary in many connections. Those discussed in some detail are listed.

(i) The measured spin and magnetic moment of F^{19} are usually explained by assuming that the odd 9th proton occupies the $2S_{1/2}$ shell. However, most nuclei seem to put their 9th to 14th nucleons into a $d_{5/2}$ shell. The $2S_{1/2}$ shell being more generally needed for only the 15th and 16th nucleons. Feenberg showed that the neutron-proton configuration ($d^2_{5/2}$; $d_{5/2}$), producing $I = \frac{1}{2}$, can explain the F^{19} magnetic moment as well as the $2S_{1/2}$ proton can. Either jj or LS coupling work equally well here. (See the Incidence of the $2S_{1/2}$ Shell.)

(j) The favored β-transitions between mirror isobars having more than one nucleon outside closed shells can be viewed as between states formed from the jj-coupling of all the extra nucleons, just as in the favored transitions of odd-odd nuclei (g), there seems to result no preference for the jj over the LS coupling. (See jj-Coupling Calculations of ft-Values.)

(k) The scatter of the favored β-half-lives has been compared against the deviations of the measured magnetic moments from the Schmidt (single-particle state) values. A strong correlation shows up when jj-coupled states of many nucleons are used as a basis of comparison between the two types of experimental data. (See ft-Values and Magnetic Moments).

(l) The sharp distinction between favored and normal allowed transitions was early explained by Wigner's super-multiplet theory. In the unfavored transitions, the two isobars are assigned different isotopic spins (T), hence isotopic spin conservation is violated.

The corresponding states were also assigned definite ordinary spins S, which is inconsistent with the large spin-orbit coupling of the shell model. Mayer asserts that the jj-coupling of several nucleons also produces states differing in T, so that the favored-unfavored distinction can be retained in the shell model. (See Isotopic Spin in the Shell Model.)

(m) Nordheim uses several-nucleon configurations to explain the unfavored β-decay of P^{30}, which was an anomaly on Wigner's supermultiplet theory (l). (See the Phosphorus Decays.)

(n) Goldhaber uses the rearrangement of several nucleons to explain the slowness of Kr^{85} β-decay between states of equal spin. Such rearrangements are characteristic of the "Class II, odd A" decays already mentioned in (c). (See Classifications of Odd A Allowed Transitions.)

(o) The participation of many nucleons in γ-radiation is apparently responsible for some of the disagreements of observed rates with expectations based on a single-particle calculation. (See The γ-Radiation.)

(p) Several-nucleon configurations are used to characterize excited states of even-even nuclei (See Even-Even Nuclei). Another approach is mentioned below (r).

The shell model was intially considered to be radically contradictory to the earlier "liquid drop" picture of the heavier nuclei. The latter nevertheless remained useful, eg., in understanding fission (See

The Liquid Drop). Now, it is supposed that the core may still behave like a liquid drop. All the distinctive shell effects are attributed to an "atmosphere" of extra-core nucleons. Moreover, core and "atmosphere" are conceived to be coupled to form the <u>collective model</u>. This implies that the core is no longer entirely inert but its excitation is important when there are many extra-core nucleons coupled to it. Only near the magic numbers and in light nuclei is the core still inert. The effects of the core excitation were discussed in several connections:

(q) Core orthogonality may contribute to the unfavored factor in heavier nuclei where T-conservation [see (i)] is not expected to hold. Nordheim asserts that this cannot be the whole story however (See The Unfavored Factor in Heavier Nuclei.)

(r) The first excited state (2 +) of even-even nuclei is supposed to represent a core excitation. Higher states arise from a superposed excitation of the extra-core nucleons. (See Even-Even Nuclei.)

(s) Enhanced electric quadrupole radiation is attributed to contributions from core oscillations (see The Coupling to Extra-Core Nucleons.)

(t) The core and the extra-core nucleons are supposed collectively responsible for the deviations of the magnetic moments from the Schmidt limits. However, an extra suppression of the anomalous single nucleon moments must also be admitted before large enough deviations can be obtained. (See Magnetic Moments.)

(u) The compound intermediate states for reactions characterizing the liquid drop model contribute heavily to stripping reactions. This contribution underlies the peaks in the angular distribution which are attributed to extra-core states. (See Stripping Reactions.)

Important for β-spectroscopy is the nature of the interaction law responsible for β-decay. The evidence for a component obeying Fermi selection rules ($\Delta I = 0$, including $0 \rightarrow 0$, in allowed transitions) in addition to the Gamow-Teller component ($\Delta I = 0, \pm 1$ but not $0 \rightarrow 0$) was discussed in some detail. The variation of the half-lives of favored transitions as well as the existence of favored $0 \rightarrow 0$ transitions provide the evidence in question (See the MIRROR NUCLEI, and Fermi Selection Rules).

Evidence that the two components have the (Scalar, Tensor) rather than (Vector, Tensor) forms was also discussed. It is based on the fact that normal once-forbidden spectra with $\Delta I = 1$ have a statistical distribution. The known large Fermi-to-Gamow-Teller ratio helps considerably in making the argument decisive. (See the Phenomenological Derivation).

The evidence that, in addition to the above two components, there is also a Pseudo-scalar component in the β-coupling has been somewhat strengthened. Originally, it was limited to the Petschek-Marshak analysis of the RaE spectrum, which made use of the amount of Pseudo-scalar interaction as an adjustable parameter. This seemed an "ad hoc" explanation of the singular spectrum, requiring, as it did, the accidentally destructive interference between the Tensor and Pseudo-scalar contributions. Brysk strengthened the case with a comparison of the interference in RaE with that in Tl^{206} which is shorter-lived. The latter nucleus has constructive interference, essentially because it has one nucleon missing from each of the $Z = 82$ and $N = 126$ shells. On the other hand, RaE has one nucleon extra to each of the same shells.

Theoretical arguments concerning the magnitude of the nuclear moment $\langle \beta \gamma_5 \rangle$, on which the Pseudo-scalar interaction acts, are in a state of flux. (See The Pseudo-Scalar Interaction in RaE.)

The complex character (Scalar, Tensor, Pseudoscalar) of the β-interaction invites a discussion as to whether it is a fundamental interaction between primary fields, or whether it is some phenomenological resultant of such interactions. Illumination can be sought through relation to meson decay, but that turns out to be too ambiguous to be helpful. (See the Universal Fermi Interaction.)

Many items discussed do not easily fit into the categories described above. They are included in the more detailed account which follows. This attempts to bring together the discussions on similar topics led by different lecturers, rather than keeping each lecturers contribution intact.

THE MIRROR NUCLEI

ft-Values and State Character

Nuclei with $N = Z \pm 1$ neutrons provide an unusual opportunity for the study of their ground states. Not only are spin (I) and magnetic moment (μ) measurement available, but also the comparative half-lives (ft-values) of their β-decay are each characteristic of essentially a single nucleon state: the parent and the daughter of a mirror pair can be presumed almost identical in state character. It is true that the $Z = N + 1$ number of the pair has extra Coulomb repulsion, providing most of the β-energy, but the effect on the distribution is negligible. This was affirmed by Mayer in answer to a query by Deutsch.

The relation of the ft-value to the matrix elements $\int 1$ and $\int \underset{\sim}{\sigma}$ of allowed β-decay is:

$$(ft)^{-1} = G_S^2 \, |\int 1|^2 + G_T^2 \, |\int \underset{\sim}{\sigma}|^2$$

where $G_{S,T}$ are, respectively, the coupling constants for β-decay under Fermi and Gamow-Teller selection rules. The Fermi matrix element $\int 1$ becomes merely a normalization integral for identical parent and daughter states, hence $|\int 1|^2 = 1$, regardless of the character of these states. It is the Gamow-Teller matrix element $\int \underset{\sim}{\sigma}$ which makes the β-decay of the mirrors sensitive to the state character.

Feenberg and Mayer discussed means of evaluating $|\int \underset{\sim}{\sigma}|^2$ for comparison with the ft-values.

jj-Coupling Calculations of ft-Values

Feenberg relied principally on calculations of $|\int \underset{\sim}{\sigma}|^2$ from assumed state characters. Wigner and others long ago supplied values for $|\int \underset{\sim}{\sigma}|^2$ which follow when the spin I of the state is built up from a specific orbital angular momentum $L = I \pm \frac{1}{2}$. Ever since the shell model indicated that large spin-orbit coupling exists, it became evident that the state may instead be the resultant of individual nucleon states of specific j-values. This jj-coupling approach coincides with the $L = I \pm \frac{1}{2}$ evaluations for (closed shell) $\pm$ (1 nucleon) cases. Feenberg supplied a list of $|\int \underset{\sim}{\sigma}|^2$ values for the other cases which can be unambiguously calculated with jj-coupling. The unambiguous cases are those for which a unique state character follows from I, parity and isotopic spins (T) conservation. The following table lists the $|\int \underset{\sim}{\sigma}|^2$ values given by Feenberg for mirror nuclei, together with the $L = I \pm \frac{1}{2}$ values for the same cases:

Transition	$I_i \rightarrow I_f$	$\lvert\int\sigma\rvert^2$ jj	$\lvert\int\sigma\rvert^2$ $L = I \pm \frac{1}{2}$	$(ft)[0.8 + \lvert\int\sigma\rvert^2]$ jj	$(ft)[0.8 + \lvert\int\sigma\rvert^2]$ $L = I \pm \frac{1}{2}$
$Be^7 \rightarrow Li^7$	$\frac{1}{2} \rightarrow \frac{1}{2}$	121/135	5/3	3900	5700
$Ne^{19} \rightarrow F^{19}$	$\frac{1}{2} \rightarrow \frac{1}{2}$	121/75	3	4200	6650
$A^{35} \rightarrow Cl^{35}$	$3/2 \rightarrow 3/2$	121/375	3/5	3800	4760
$K^{37} \rightarrow A^{37}$	$3/2 \rightarrow 3/2$	121/375	3/5	3080	3850

Depending on which type of coupling is the more correct, one of the last two columns should be constant, as can be seen from the above relation to the ft-value. The numeric 0.8 is the value adopted for G_s^2/G_T^2 (see just below and also the β-DECAY LAW). The value of the constant in the last columns should be about 4800, according to the one-particle mirror transitions. The latter, however, do not actually yield greater constancy at 4800 than shown in the last two columns of the table here.

The Incidence of the Second S-Shell

The F^{19} state is usually presumed to be the 1-particle $S_{1/2}$ state because of its measured spin $I = \frac{1}{2}$. Feenberg stressed that it is unnecessary to accept this intrusion of an $S_{1/2}$ state in a region where $d_{5/2}$ nucleons are otherwise expected. Feenberg uses for F^{19} the 2 neutron, 1 proton configuration: $(d_{5/2}^2 ; d_{5/2})_{1/2}$, with isotopic spin $T = \frac{1}{2}$. LS coupling works equally well.

Here, an objection by Mayer should be interpolated. The jj-coupled $T = \frac{1}{2}$ state should actually be expected to lie higher than other states of the same configuration (see Isotopic Spin in the Shell Model). The calculations are admittedly crude, but accepting Feenberg's assignment for F^{19} forces one to suppose that level positions can be greatly altered by subtler effects than such as can be estimated at present.

Feenberg discussed further the evidence against the assignment of $S_{1/2}$ to the 9th and 10th nucleons. The measured magnetic moment of F^{19}, $\mu = 2.63$ nuclear magnetons, is consistent with an $S_{1/2}$ proton ($\mu_p = 2.79$) but the configuration he adopts works as well, yielding $\mu = 2.75$. Further, if the 10th nucleon fills an $S_{1/2}$ shell, Na^{23} would have only one d proton instead of the $(d_{5/2}^3)_{3/2}$ proton configuration usually regarded as more consistent with the evidence. (It is none too clear, however, that a filled S shell plus the two neutrons, 1 proton configuration $(d_{5/2}^2 ; d_{5/2})_{3/2}$ would not explain Na^{23} as well. A clearer case should be provided by Ne^{21}, which would have no extra protons if a filled S-shell intruded. However, its spin is unmeasured; it is usually guessed as $I = 3/2$ only because the odd neutron is the 11th, and Na^{23} has a measured $I = 3/2$ with an odd 11th proton. The Ne^{21} magnetic moment is only known to be negative, and this is irreconcilable with a $d_{3/2}$ neutron, according to the Schmidt diagram. It would help to eliminate the S-shell and thus have a $(d_{5/2}^{3})_{3/2}$ neutron configuration from which to get the negative moment.

ft-Values and Magnetic Moments

Mayer pointed out that when the experimental $(ft)^{-1}$ values are plotted against well-calculated values of $|\int\sigma|^2$, the above relation between these quantities should make the points fall on a straight line, with intercept G_S^2 and slope G_T^2. Unfortunately, as the above discussion indicates, the calculated $|\int\sigma|^2$ values are not very reliable. Mayer therefore uses "measured" $|\int\sigma|^2$ values, i.e. values implied by measured magnetic moments and, particularly, their deviations from the Schmidt limits.

Correlation between ft-values and magnetic moments have been attempted before, eg. by Kofoed-Hansen, and by Feenberg and Trigg. Mayer's approach follows from just the essentials of the jj-coupling of all the nucleons.

Mayer illustrated the existence of an empirical correlation by comparing the proton and P^{31}. The latter clearly has its odd proton in an $S_{1/2}$ state like that of the isolated proton. Both are consequently expected to have $|\int\sigma|^2 \approx 3$ on any coupling scheme. Yet $(ft)^{-1} = 7.2\ (10)^{-4}$ for the neutron to proton decay, whereas $ft = 2.9\ (10)^{-4}$ for $S^{31} \to P^{31}$. The discrepancy by a factor 2.5 closely parallels the discrepancy in magnetic moments: 1.3 n.m for P^{31} vs. $\mu_p = 2.79$ n.m. for the proton.

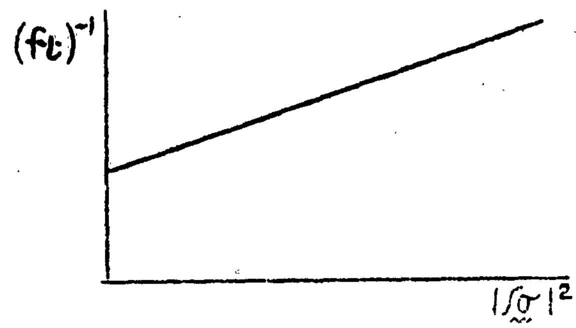
A close relation between $|\int\sigma|^2$ and the magnetic moment is to be expected theoretically, when $|\int\sigma|^2$ connects parent and daughter states as identical as in the mirror pairs. Kofoed-Hansen had written down the relation between μ and $\int\sigma_z$ for a 1-particle state with angular momentum j; it is

$$\mu = j + \tfrac{1}{2} + (\mu_p - \tfrac{1}{2}) \int\sigma_z$$

when the single particle is a proton. This gives only one component of $\int\sigma$ but that is sufficient since the relation $|\int\sigma|^2 = (I+1)/I \cdot |\int\sigma_z|^2$ can be proved. Kofoed-Hansen's restriction to single-particle states is unsafe for the empirical comparison of μ and $\int\sigma$ because μ rarely lies on the Schmidt lines.

Mayer was able to show that the relation between μ and $\int\sigma_z$ remains nearly as simple as Kofoed-Hansen's when all the nucleons are taken to participate in forming μ and $\int\sigma_z$. Of course, a specific theoretical calculation of μ or $\int\sigma_z$ would usually require falling back to the single-particle states, but that is unnecessary when experimental μ and ft-values are to be compared. Feenberg interpolated the remark, that Kofoed-Hansen's formulae can also apply to a species of many-particle states. They remain unchanged if one takes all particles outside closed shells into account and then adopts only doublet states (for odd A nuclei, of course).

Mayer exhibited a plot of $(ft)^{-1}$, against $|\int\sigma|^2$ as deduced from the magnetic moments according to the relations she found. It had the expected general appearance



with the actual points surprisingly little scattered about the straight line drawn freely through them. Thus, the close relation between the scatter of the mirror ft-values, with the deviations of μ from the Schmidt limit, appears to be substantiated.

Talmi raised an objection to the exclusive reliance on jj-coupling in Mayer's procedure. That approach does little better than LS coupling in reducing the size of matrix elements, as required by the small magnetic moment and large ft-value of P^{31}, for example. This objection refers strictly to calculated values, but makes using the jj coupling as a basis for empirical interpolation somewhat questionable. Moreover, Talmi emphasized that the LS coupling may well be needed to resolve other situations. A concrete instance was mentioned by Nordheim in answer to queries about the large ft-value of C^{14}. The best explanation for the latter anomaly still seems to be that the transition is from an S state in C^{14} to a very pure D state of N^{14}, easily explainable only if LS coupling prevails in that state. Feenberg objected to this as a final conclusion since jj coupling gives rise to mixtures of S and P states which might give the small transition probability with a suitable choice of phases. However, Mayer pointed out that the phases derived from a straightforward application of jj coupling are not suitable for the purpose.

The intercept and slope of the straight line depicted above lead to roughly $G_S^2/G_T^2 \approx 1$. Feenberg reported that closer examination makes $G_S^2/G_T^2 \approx 0.8$ the most probable value. In any case, the non-vanishing intercept of Mayer's straight line plot is one way of making explicit the evidence of the Mirror nuclei that part of the β-interaction is subject to Fermi selection rulse (see The β-Decay Law).

Magnetic Moments of Mirror Pairs

Feenberg made an additional contribution to the discussion of the mirror nuclei. He pointed out that if the magnetic moments, μ_N and μ_Z, of both members of a mirror pair with spin I are measured, then $\mu_N \pm \mu_Z$ are amenable to simple theoretical interpretations. On the basis of LS coupling in doublet states,

$$\mu_N + \mu_Z = \mu_I^m + \mu_I^p$$

where $\mu_I^{m,p}$ are magnetic moments of a single nucleon with total angular momentum I. The jj coupling treatment of the doublet state yields

$$\mu_N + \mu_Z = (\mu_j^m + \mu_j^p)\,I/j$$

where j is the value for the odd nucleon. In earlier work, Trigg had mixed $L = I \pm \frac{1}{2}$ states of equal parity, to account for deviations from the Schmidt limits. This approach makes values for $|\int\sigma|^2$ deducible from $\mu_N + \mu_Z$. When modified to conform to jj-coupling, an expression for $|\int\sigma|^2$ proportional to $(\mu_N - \mu_Z)^2$ is obtainable.

FAVORED VS. NORMAL ALLOWED β-TRANSITIONS

The Problem of Favored vs. Unfavored Transitions

The mirror nuclei display another aspect: they supply most of the cases of favored β-decay. Such cases form a well-defined group of anomalously short comparative half-lives: ft = (0.85 to 4.7) 10^3 sec vs. 10^{4-6} sec for normal allowed transitions. Besides the mirrors, only members of isobaric triads ($A = 4n+2$) undergo favored decay.

Wigner's supermultiplet theory early gave a neat explanation of just why mirrors and isobaric triads, and no other nuclei, should undergo favored transitions. This theory assumes isotopic spin (T) conservation, and also the approximate conservation of the total ordinary spin (S). The last assumption became questionable after the advent of the shell model, with its indications of a strong spin-orbit coupling.

The shell model, or any other theory which predicts identical states for mirrors and isobaric triads, accounts for favored decay in those nuclei. The difficulty with the shell model is that it provides nominally identical states also for many isobars whose transitions are clearly unfavored. One of many examples is $S^{35} \rightarrow Cl^{35}$, quoted by Nordheim. It has ft = 1.6 $(10)^5$ sec and appears to transform a $d^3_{3/2}$ neutron configuration into $d^2_{3/2}; d_{3/2}$. On pure shell model considerations, one may compare it to He^6 which undergoes $p^2_{3/2} \rightarrow p_{3/2}; p_{3/2}$ and is distinctly favored (ft = 850 sec.) The question of how the shell model might cope with the distinction between favored and normal allowed transitions was discussed by Nordheim, Mayer and Feenberg.

The jj Coupling Results

Feenberg expanded the table shown in the preceding section to include the isobaric triad transition for which $|\int \sigma|^2$ vanishes because of the spin change:

Transition	$I_i \rightarrow I_f$	$\lvert\int\sigma\rvert^2$ jj	$\lvert\int\sigma\rvert^2$ LS	$(ft)\lvert\int\sigma\rvert^2$ jj	$(ft)\lvert\int\sigma\rvert^2$ LS
$He^6 \rightarrow Li^6$	$0 \rightarrow 1$	10/3	6	2830	5100
$C^{10} \rightarrow B^{10*}$	$0 \rightarrow 1$	10/3	6	5670	10200
$C^{14} \rightarrow N^{14}$	$0 \rightarrow 1$	2/3	6	$7(10)^8$	$6(10)^9$
$F^{18} \rightarrow O^{18}$	$1 \rightarrow 0$	14/15	2	3400	7800
$Al^{26} \rightarrow Mg^{26*}$	$1 \rightarrow 0$	14/15	2	2000	4400

*It has long been considered likely that the Al^{26} transition is to an excited state of Mg^{26} although no γ-ray has been reported. This was because the β-energy was considerably less than the energy available according to semi-empirical mass formulae. Moreover, the ground state of Mg^{26} is expected to have a large spin by Nordheim's rules, hence the transition to it should be forbidden. However, Goldhaber called attention

As in the table of the last section, the values in the last two columns are to be compared to 4800, which is the value expected from the more straightforward single particle mirror transitions. The reporter calls attention to the apparent fact that the LS coupling results seem to show to slightly better advantage here, than do the jj-coupling results. Too large values of $ft\,|\int\underset{\sim}{\sigma}|^2$ are somewhat more easily accounted for than too small ones (see eg., Classification of Odd-A Allowed Transitions). The C^{14} case was already discussed near the end of the preceding section, also as more easily understood with LS coupling. Feenberg did stress that the supermultiplet theory (which implies LS coupling) has been too successful to be entirely discarded. He suggested that some form of intermediate coupling may eventually resolve the contradictions: eg., spin-orbit forces may still be weak compared to the forces between nucleons in equivalent orbits.

Feenberg went on to present results for a few unfavored transitions:

Transition	$I_i \rightarrow I_f$	$\lvert\int\underset{\sim}{\sigma}\rvert^2(jj)$	ft	ft $\lvert\int\underset{\sim}{\sigma}\rvert^2$
$B,N^{12} \rightarrow C^{12}$	$1 \rightarrow 0$	16/9	$1.3(10)^4$	$2.5(10)^4$
$O^{19} \rightarrow F^{19}$(gd.)	$3/2 \rightarrow 1/2$	4/5	$3.5(10)^5$	$2.8(10)^5$
$S^{35} \rightarrow Cl^{35}$	$3/2 \rightarrow 3/2$	4/25	10^5	$1.6(10)^4$

It should be noted that the Fermi matrix element vanishes in the $d^3_{3/2} \rightarrow d^2_{3/2}\,;\, d_{3/2}$ transition of S^{35} (see above) even though there is no spin change. Of course, the jj-coupling matrix elements are not small enough to bring the values in the last dolumn down to the favored norm, 4800. (See remarks by Talmi in THE MIRROR NUCLEI). Thus, the table reiterates the difficulty pointed out above, that the usual applications of the shell model predict favored magnitudes for transitions which empirically are definitely unfavored.

The Phosphrous Decays

Nordheim suggested the B^{12}, N^{12} decays to C^{12} as prototypes of empirical unfavored transitions. Both transform $(p_{3/2}{}^3;\, p_{1/2})$ configurations into the filled shell $(p_{3/2}{}^4)$ structure of C^{12}.[1] The pattern here led Nordheim to an explanation for the empirically unfavored P^{30} decay (ft = 10^5sec.). This case was the single, specific failure of the supermultiplet theory, which predicts favored decay for the presumed transformation, $(S_{1/2};\, S_{1/2}) \rightarrow S_{1/2}{}^2$. Nordheim suggests that the $S_{1/2}{}^2$ neutron shell is already full in the parent, as in the daughter.[1/2] Instead, a $d_{5/2}$ neutron is missing, making the neutron configuration: $d_{5/2}{}^5\, S_{1/2}{}^2$. He hypothesizes a $d^6_{5/2}\, d_{3/2}$ proton configuration, in order to obtain a $d_{3/2} \rightarrow d_{5/2}$ transition analogous

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to a recent report by Kay Way that revised mass formulae no longer allow for an excess over the observed β-energy release.

to the $p_{1/2} \rightarrow p_{3/2}$ in B^{12}. Nordheim asserted that also other evidence points to an extra stability of odd neutron-odd proton pairs $\ell + \frac{1}{2}; \ell - \frac{1}{2}$ (See Classification of Even - A Allowed Transitions). The unfavored decay of P34 again follows the $d_{3/2} \rightarrow d_{5/2}$ pattern on the basis of the configurations $(d^5_{5/2}; d^3_{3/2})^{3/2} \rightarrow (^{5/2}d^6_{5/2}; d^2_{3/2})$. Nordheim does not dissolve the $d^6_{5/2}$ proton shell in favor of $S^2_{1/2}$, as he did the neutron shell of P30. A case in point is P32, in which he retains a single $S_{1/2}$ proton, together with a $S^2_{1/2} d_{3/2}$ neutron configuration, outside filled $d^6_{5/2}$ shells. (Accordingly, he still relies on the ℓ-forbidden $d_{3/2} \rightarrow S_{1/2}$ transition to explain the very long comparative half-life of P32.) The reason for treating the protons differently from the neutrons here, is that protons may go more easily into d than S orbits because of coulomb repulsion. Thus, in this first part of Nordheim's discussion, some features of the shell picture of unfavored transitions were exhibited and on its basis a solution for the one failure of the supermultiplet theory could be suggested.

Isotopic Spin in the Shell Model

Mayer considered it possible to account for the favored-unfavored distinction on the basis of the shell model, supplemented only with the requirement of isotopic spin conservation. A configuration of more than two like nucleons is always involved in unfavored transitions. When jj coupling is used to construct the consequent states, the one expected to fall lowest in energy is different in T-value for the two isobars connected by β-decay. Hence, the transition between such states is partially forbidden by the approximate T-conservation. Thus the reasoning is much the same as in Wigner's supermultiplet theory, except that one applies jj coupling to nucleons outside closed shells, in place of LS coupling to nucleons outside saturated cores of $4n$ nucleons.

Axel interposed the comment that favored transitions to excited states should be expected, since levels of proper T-value for conservation must exist in the above picture. Mayer replied that such excited states should be expected to lie very high, inaccessible for the energy made available by β-decay.

Mayer supported her explanation of the unfavored transitions by extending the $(ft)^{-1}$ vs. $|\int\sigma|^2$ plot, depicted in the preceding section, to include unfavored decays. The latter are now expected to connect orthogonal states, hence the Fermi matrix element, $\int 1$, vanishes. The operation σ in $|\int\sigma|^2$ leads to contributions $(ft)^{-1} = G_T^2 |\int\sigma|^2$ hence a straight line plot of zero intercept is to be expected for unfavored decay. Points obtained with $|\int\sigma|^2$ values deduced from magnetic moments were found to cluster near the origin of the plot. Thus the expectation of a zero intercept seems to be borne out. The small values of $|\int\sigma|^2$ indicated by the magnetic moments are in some accord with the smallness they exhibit in the unfavored β-transitions.

The "Unfavored Factor" for Heavier Nuclei

Nordheim pointed out that isotopic spin conservation can be expected to account for unfavored decay only in the lighter nuclei. The large coulomb forces in heavier nuclei destroy the charge symmetry of the forces and T can no longer be a good quantum number. Yet the "unfavored factor", by which normal allowed transitions are slower than favored ones, persists with about the same strength for heavier nuclei. He presented the problem as one concerned with the size of β-matrix elements like

$$\int \psi_f^* \mathcal{O} \psi_i \cdot \int \underline{\psi}_f^* \underline{\psi}_i$$

where $\mathcal{O}$ is the β-decay interaction operator, while i and f refer to initial and final nucleus, respectively. The ψ's describe nucleons outside closed shells while the $\underline{\psi}$'s characterize the filled shell core. The first integral factor is to be held responsible for the large differences between different types of β-transitions. Work like that of Brysk, which utilizes one particle states for ψ, seems to be in fair accord with the ratios of different types of transitions. However, there appears to be the additional "unfavored factor" common to all transitions except the "favored" ones. One might consider attributing it to the second integral factor, i.e. to "core orthogonality." This probably contributes to the observed phenomena. However, it might then seem reasonable that the "unfavored factor" show a trend with the mass number, A. However, Nordheim's studies of the evidence failed to disclose any such trend. Another expectation follows from Bohr's conception of the core: it should be most deformable when there are many nucleons in the unfilled shells. Again, no such correlation turns up. It appears that the "unfavored factor" in heavier nuclei is not yet completely understood.

STUDIES OF COMPARATIVE HALF-LIVES (ft)

Shell Model Classifications of ft-Values

The shell model assignments of nuclear states have made it possible to classify β-transitions reliably into the various types of allowed and forbidden decay. Now the ft-values fall into fairly well-defined groups, although there still remains a good deal of scatter within each group. Nordheim described a series of studies he undertook in an attempt to subdivide the groups further and thus reduce the scatter.

First Nordheim could find no discernably systematic difference in ft-values of allowed transitions with no spin change from those with $\Delta I = 1$. (See a $\Delta I = 0$ and 1 difference found for once-forbidden transitions, below.).

Classification of odd-A Allowed Transitions

Next, he did discover a significant way to separate all normal allowed transitions of odd mass number A into two classes. He defines Class I transitions as those in which the nucleonic configurations are unchanged except for the last odd nucleon in each of the β-isobars. An example is

$$\text{Class I: } {}_{42}Br^{77}_{35} \rightarrow {}_{43}Se^{77}_{34} + \beta^+$$

in which the odd 35th proton ($p_{3/2}$) is transformed into an odd 43rd neutron ($p_{1/2}$). Class II consists of "rearrangement" transitions. Pairs of nucleons in equivalent orbits must be dissolved and new pairs formed, as in:

$$\text{Class II: } {}_{44}As^{77}_{33} \rightarrow {}_{43}Se^{77}_{34} + \beta^-$$

As^{77} has two more neutrons than Br^{77}, part of a $g_{9/2}^{\,6}$ group. This group must be dissolved when a neutron is transformed into a proton, to leave the $g_{9/2}^{\,4}\, p_{1/2}$ neutrons characterizing Se^{77}. The new proton must then pair with the odd $p_{3/2}$ proton of As to form the even proton combination of Se. All this rearrangements leads to an As transition which is 5 times slower (in ft-value) than is the Br decay.

Nordheim exhibited a graph of 22 Class I ft-values and 18 Class II points, plotted against A. A general settling of the Class I ft-values below those of Class II was clearly discernable, the difference being about a factor 4 to 5 in ft. Only one (Ti^{45}) Class II point had ft $< 10^5$ sec., whereas 12 Class I points lay below this value. Considering the possible incidence of other slowing down factors, such as ℓ-forbiddeness and core orthogonality, some unusually high ft-values should also be expected in Class I. The significance is in how, practically without exception, no low ft-values occur for Class II transitions. The reliability of the "rearrangement" slowing down factor, 4 to 6, must be judged in relation to about a factor 2 of error which may be due to the inaccuracy of energy and branching ratio measurements.

Goldhaber discussed a case discovered earlier in which rearrangement appears to cause a spectacular delay in β-radiation, by a factor $\sim 10^4$. This is the transition from Kr^{85} to a second excited state in Rb^{85} which has ft $\approx 10^9$ sec in spite of an apparently allowed spin change $\Delta I = 0$, with no parity change. The crucial point is the assignment of a $g_{9/2}$ character to Rb^{85**}. This appears to be proved by its magnetic quadrupole γ-transition to the well fixed $f_{5/2}$ ground state. Allowed K-capture from the $g_{9/2}$ state of Sr^{85} seems to confirm the assignment. Goldhaber supposes the excited Rb state to represent excitation of the proton from $f_{5/2}$ (ground) to $g_{9/2}$. An intermediate Rb^{85*} state represents the expected intermediate excitation to $p_{3/2}$. To account for the slowness of the decay of the $g_{9/2}$ neutron in Kr to the $g_{9/2}$ proton in Rb, a rearrangement of the neutrons is hypothesized. In Kr, 11 neutrons are distributed between the $p_{1/2}$ and $g_{9/2}$ orbits, filling them except for one $g_{9/2}$ place. In Rb, there 10 neutrons, just enough to fill the $g_{9/2}$ orbits, leaving $p_{1/2}$ empty. The close $p_{1/2} - g_{9/2}$ competition in the filling up to magic number 50 is well-known; it accounts for a large class of isomers (including Kr^{85}, and Sr^{85}). The preference for ${g_{9/2}}^{10}$, instead of ${g_{9/2}}^{8}\,{p_{1/2}}^{2}$, is plausible: Goldhaber refers to it as a stabilization of $g_{9/2}$ neutrons by the presence of the $g_{9/2}$ proton. Hence, in the Kr to Rb transition, after the $g_{9/2}$ neutron is transformed into a proton, the remaining ${g_{9/2}}^{8} + {p_{1/2}}^{2}$ neutrons must be rearranged into a ${g_{9/2}}^{10}$ group, causing delay of the β-radiation.

Classification of Even-A Allowed Transitions

Since even A transitions always involve an even-even nucleus, there seems to be no chance here for class distinction, in the above sense. Nevertheless, Nordheim again found a significant distinction between two classes. Allowed transitions probably must transform nucleons of a given ℓ-value. The last neutron-proton pair of the odd-odd β-isobar may have the pair if j - values $\ell+\frac{1}{2}\,;\ \ell-\frac{1}{2}$ which Nordheim uses to characterize the Class I of even A nuclei. Class II is to have $\ell+\frac{1}{2}\,;\ \ell+\frac{1}{2}$ or $\ell-\frac{1}{2}\,;\ \ell-\frac{1}{2}$ instead. The latter configurations should all lead to a high spin I for the odd-odd nuclear ground state, according to the long known "Nordheim rules." This is impressively confirmed when all the allowed even A transitions are considered. 18 of 22 Class II transitions are forced to go to excited states because of the 0 spin of the even-even ground state. The three exceptions are Ga^{68} and the two Cu^{64} transitions in which the parent spin is only I = 1, due to $(p_{3/2};\ p_{3/2})_1$ configurations. The Class II transitions to excited states all have log ft > 4.8, with two exceptions; 18 of the 22 have log ft > 5. On the other hand, 18 of 26 Class I (even A) transitions have log ft < 5! The low ft-values of the latter even A nuclei had been noticed before, by Feenberg and Trigg. Their Class I characterization offers a welcome, unforced explanation: The β-transition requires only a spin-flip which way makes it unfavored (See Favored vs. Normal Allowed Transitions) but not slowed down by rearrangement. The paired neutron-proton of Class I is apparently favored in energy also. For example, $(g_{9/2}; g_{7/2})_1$ ground-states occur beyond the range in which these orbitals are found in odd A nuclear ground states.

Classification of Once-Forbidden Transitions

Nordheim went on to study the once-forbidden transitions with $\Delta I = 0$ or 1. (but change of parity). Here he found a significant difference of ft-values between $\Delta I = 0$ and $\Delta I = 1$ cases. 13 of 15 $\Delta I = 1$ cases have $7.2 \leq \log ft \leq 7.7$. The other 2 have log ft = 6,9, 7.1. Almost all the 18 $\Delta I = 0$ ft-values lie lower, and moreover show a discernably systematic decrease when plotted against A. For $A < 150$, the $\Delta I = 0$ cases have $6 < \log ft < 7.3$. For $A > 150$, $\log ft < 6.2$. Hg^{205}, Tl^{206} and Pb^{209}, with their extraordinarily low log ft = 5.2 to 5.5, appear to be extrapolations of the trend. Goldhaber, and also Konopinski and Langer, had ascribed the latter cases to the fact that both neutron and proton shells are nearly closed (at $Z = 82$, $N = 126$). This may still be a contributing factor, and the secular decrease of the $\Delta I = 0$ ft-values with A not quite as striking as Nordheim's diagram shows.

Nordheim pointed out that in once-forbidden transitions a new possibility for $\Delta I = 0$ decay arises, which may account for the fact that the latter has lower ft-values than $\Delta I = 1$, whereas a similar distinction could not be found for allowed transitions (see above). The pseudo-scalar (P) -interaction, if it exists, would contribute for the first time to the $\Delta I = 0$ transitions with parity change. This possibility is related to the singular RaE case.

The peculiar RaE spectrum (see The β-Decay Law) was fitted by Marshak and Petschek using the amount of P-interaction as an adjustable parameter. Konopinski and Langer pointed out the reason that this did not lead to a statistical shape, as it usually does when a large Coulomb energy is involved. An accidental cancellation of P and T (tensor) interaction contributions was tacitly admitted by the Marshak-Petschek analysis and this not only gives the singular spectrum shape but also implies a considerable slowing down of the transition. Thus the large ft-value (10^8 sec) of the presumed $0 \rightarrow 0$ transition of RaE is accounted for. The slowing involved is particularly striking when one notices that the otherwise expected ft-value against which RaE should be compared is the anomolously low one of the group indicated above: Hg^{205}, Tl^{206} and Pb^{209}.

Nordheim suggested that the ThB and ThC decays are slowed down somewhat by the same effect as that shown in RaE. This presumes the assignment 0^+ for the ThC ground state, which is the daughter of ThB, and the parent of a transition to ThC'. However, Sherr reported evidence, found by Horton of Princeton, that the ThC ground state is actually 1^-. This was based on $\alpha\gamma$ correlations in the competing α-decay of ThC to ThC''. Thus, the slowness (log ft = 6.8, 7.2) of the ThB, C decays is better ascribed to a $\Delta I = 1$ spin change.

THE LAW OF β-DECAY

Fermi Selection Rules

It has long been known that the β-interaction must include a component leading to Gamow-Teller selection rules. The evidence for this is listed as Argument I in the recent Mahmoud-Konopinski "phenomenological derivation " of the β-decay law. Their argument II consisted of the evidence that also to be included is a component giving Fermi selection rules. Part of that evidence comes from the analysis of mirror transitions (see THE MIRROR NUCLEI). Somewhat more direct evidence is the existence of short-lived $0^+ \rightarrow 0^+$ transitions. Such would be twice-forbidden, perhaps a factor 10^6 longer-lived, if only Gamow-Teller rules prevailed.

Sherr discussed the evidence for the existence of the $0 \rightarrow 0$ transitions in O^{14} and C^{10}, due mostly to himself and co-workers at Princeton.

The first case found was the $O^{14} \rightarrow N^{14*}$ transition to the first excited state of N^{14}. The transition to the ground state is probably ℓ-forbidden like that of the low energy C^{14} decay (see THE MIRROR NUCLEI).

The assignment 0^+ to the N^{14} state excited by the β-transition is based on its coincidence in excitation energy with a state expected to occur in N^{14} which is analogous to the C^{14} and O^{14} ground states (each 0^+). Feenberg's calculated value for the excitation energy, 2.31 Mev, is to be compared to the energy now measured for the γ-ray which follows the β-emission: 2.30 ± 0.03 Mev. Sherr mentioned important supporting evidence for the interpretation: the state in question has failed of excitation by the inelastic scattering of deuterons and alpha particles on N^{14}, although it has appeared in the $C^{13}(d,n)N^{14}$ reaction. A failure to excite the state of N^{14} analogous to C^{14} and O^{14}, by means of $N^{14} + d$ or α, is expected on the basis of charge-symmetric internucleon forces.

The $C^{10} \rightarrow B^{10}$ transition is complicated by the known existence of three energetically accessible excited states of B^{10}: 0.72, 1.74 and 2.15 Mev above ground. Moreover, the identification of the one analogous to the C^{10}, Be^{10} ground states (0^+) is not as easy as in O^{14} because the calculations are much more uncertain: The Coulomb exchange integrals are more important here. The uncertain calculations give 1.9 Mev for the excitation of the 0^+ state. Thus, it may be identified with either the 1.74 Mev or the 2.15 Mev state. The occurrence of the intermediate 0.72 Mev state is not unexpected; since the B^{10} ground state has $I = 3$, there is room for a 1^+ state. The short life of the β-transitions to the 0.72 Mev state further indicates the assignment 1^+ ; finally, the observed promptness of the 0.72 Mev γ-ray shows that it cannot be nearly as slow as $0 \rightarrow 3$ radiation would be.

Sherr and Gerhart find two γ-rays following the β-transition: 0.723 ± 0.015 and $1.033 \pm .030$ Mev ($\approx 1.74 - 0.72$ Mev). No 1.43 Mev (2.15-0.72) nor 2.15 Mev γ-radiation could be found, a limit of 1 in a 100 to 1000 β-transitions being set. Of course, that would not be inconsistent with a 0^+ 2.15 Mev state, if no Fermi rules applied.

Data from published particle reaction experiments is called on to help show that the 1.74 Mev state is indeed the 0^+ analogue of C^{10} and Be^{10}. It is found that all the above B^{10} states are excited by protons on B^{10} but the 1.74 Mev state cannot be excited by deuterons or α-particles on B^{10}. This is just what is to be expected for the analogue state, on the basis of charge symmetric internucleonic forces. Further, one can compare the intensities of γ-radiation from the three levels when excited in the $Be^9(d,n)\,B^{10}$ reaction. The theoretical expectations (based on Weiskopf's γ-lifetime formulas) are too uncertain to prove more than just: I(1.74 Mev) = 0 or 1; I (2.15 Mev) = 1 or 2. Thus, only the 1.74 Mev state is left to be identified as the analogue state. (if the assignemtn 0^+ is accepted for it, then I = 1 in the 2.15 Mev state.)

Finally, the measured intensity of the 1.033 Mev γ-ray is found to be $1.65 \pm 0.2\%$ of the 72 Kev γ-intensity. This is only within the limits expected for a β-transition to the 1.74 Mev state which is as favored as that to the 72 Kev state. Hence, it is clear that one has to do with another favored $0 \rightarrow 0$ transition, possible only with Fermi selection rules.

Sherr went on to present the result for the ratio of the β-coupling constants (see THE MIRROR NUCLEI and also below) which follows from the relative C^{10} β-decay intensities. $G_S^2/G_T^2 = 0.8$ (0.66 to 1.06) if the β-matrix elements are evaluated with LS coupled nuclear states ($|\int\sigma|^2 = 6$ for the transition to the 72 Kev state). $G_S^2/G_T^2 = 0.44$ (.37 to .6) if evaluated with jj coupling ($|\int\sigma|^2 = 10/3$, see Feenberg's table in the Favored vs. Unfavored Transitions). The first evaluation is in better accord with that obtained from the analysis of the mirror transitions (see THE MIRROR NUCLEI).

The O^{14} data was already used in conjunction with the H^3 decay in Blatt's evaluation:

$$G_S^2/G_T^2 = 0.54 \text{ (0.3 to 1)}.$$

<u>The Phenomenological Derivation</u>

<u>Konopinski</u> discussed the arguments which lead to an STP combination as the correct form of the β-coupling. The letters X = S, V, T, A, P, are used as symbols for the scalar, vector, tensor, axial vector and pseudo-scalar forms, respectively. If G_X is the Fermi coupling constant measuring the strength of the X-interaction, $\Sigma G_X X$ is the general expression for the interaction energy density under the essential criteria of the Fermi Thoery. By a "phenomenological derivation" is meant a determination from the evidence of which of the G_X must vanish and which not.

The first two steps of the derivation were mentioned above:

I G_T or $G_A \neq 0$ to obtain Gamow-Teller selection rules.

II G_S or $G_V \neq 0$ to obtain Fermi selection rules.

Since the Mahmoud-Konopinski publication, a stronger version of I has become available :

Ia $G_T \neq 0$ to obtain the β-recoil angular correlation observed for He^6.

The expected β-neutrino correlation is $1 \pm \frac{1}{3}\frac{v}{c}\cos\vartheta$ for the T and A interactions respectively. He^6, with $\Delta I = 1$, can decay with the short life observed only under Ganow-Teller selection rules.

Decisive results are now supplied by the angular correlation experiments because of the improved detection of the Li^6 recoils. This was achieved both at Illinois, by Allen and Jentschke, and at Columbia, by Ruby and Rustad.

A parallel strengthening of II should be obtainable from β-recoil measurements on O^{14}. This $0 \rightarrow 0$ decay (see preceding section) can result only from S or V interactions. The expected β-neutrino correlations are then $1 \pm (v/c)\cos\vartheta$.

There is one piece of evidence that the P interaction must also be included in the β-coupling:

III $G_P \neq 0$ together with $G_T \neq 0$, to obtain the singular RaE spectrum shape.

The Petschek-Marshak analysis of RaE is discussed in the next section.

Arguments Ia, II and III all require including another component in the β-law, none excludes a considerable admixture of other components. Arguments for the <u>exclusion</u> of certain combinations are based on the fact that interference between them can lead to distortions of the spectra.

IV G_S <u>or</u> $G_V = 0$ to exclude deviations of the allowed spectra from the statistical shape.

and G_T <u>or</u> $G_A = 0$

The limits set by Mahmoud from studies of the reportedly statistical shapes of the N^{13}, Cu^{64} ($\pm$) and S^{35} spectra are: $G_X^2/G_{X'}^2$ not larger than about 1%, if X, X' are either of the combinations in question. The results have been essentially confirmed by others. The smallness of the mixture permitted makes it difficult to defend any theory calling for such a mixture.

The arguments I–IV leave only arbitrary combinations STP and VTP as alternatives. Mahmoud and Konopinski developed an argument V to choose between these. They also used it against SA combinations before Ia made that unnecessary.

Argument V develops when one attempts to account for the statistical shapes observed for once-forbidden spectra with $\Delta I = 1$. The necessary condition for this can be expressed most simply as: Coulomb energy at the nuclear radius $\gg$ kinetic energies of the electron and neutrino. It is however a sufficient condition only if:

V $\quad G_V$ or $G_T = 0$ $\qquad$ to obtain statistical shapes for once-forbidden

and $\quad G_S$ or $G_A = 0$ $\qquad$ spectra with $\Delta I = 0, 1$.

and $\quad G_A$ or $G_P = 0$

The last (AP) of these is not substantiated because of the lack of unobscured once-forbidden spectra with $\Delta I = 0$. The deviations from the statistical shape expected from the VT, SA or AP interferences are of the same "Fierz" type as the theoretical deviations in allowed spectra discussed under IV. Their theoretical existence had not been clear originally because of the many effects which contribute to a once-forbidden transition.

The evidence against the existence of "Fierz-type" interference in once-forbidden spectra is in one respect clearer than for allowed spectra. When one tries to employ the VTP β-law, then the known mixture of Fermi and Gamow-Teller rules makes $G_V^2/G_T^2 > 30\%$. (See preceding section.) Mahmoud's analyses of the observed Pm^{147}, W^{185} and Pr^{143} spectra indicate $G_V^2/G_T^2 \lesssim 1\%$.

Deutsch called attention to the fact that certain possible contributions to the once-forbidden spectra with $\Delta I = 1$ seem to be ignored in these analyses. The particular contribution in question is the same one which is solely responsible for the "unique-forbidden" spectra ($\Delta I = 2$), and therefore would lead to deviations from the statistical shape. Mahmoud and Konopinski argued that this term is as negligible as the others which are dropped when a large Coulomb energy exists. This is supported by the fact that unique-forbidden transitions are factors $10^{2 \text{ to } 4}$ slower than the $\Delta I = 1$ (parity change) transitions.

On the other hand, as Deutsch pointed out, the ignored term does seem to play a leading part in many $\beta - \gamma$ correlations, although the β-spectrum gives a linear Fermi plot. However, the shapes measured in coincidence with γ-rays are not yet completely reliable.

The result of the arguments should be expressed as the

ST(P) - law

of β-decay, since only one piece of evidence calls for inclusion of the P form (see, however, Nordheim's classification of Once-Forbidden Transitions). Of great interest is the accumulating evidence that the Critchfield-Wigner S-A-P law cannot be correct (see below). The latter law has fixed relative sizes for its components and also fixed phases. This is because it was the result of theoretical hypothesis. The empirical ST(P)-Law is as yet undetermined as to relative phases, while relative component sizes are known only roughly (see preceding section). An indication that the relative TP phases can be expressed by T + P, in the conventional definition of these forms, was discussed by Brysk (see next section).

The Pseudo-scalar Interactions in RaE

The only datum requiring a P-component of β-interaction for its explanation is the spectrum of RaE. Petschek and Marshak showed that it can at present be understood only as the result of a destructive interference between P and T contributions. The support to this analysis given by the comparative half-life was discussed (see Classification of Once-Forbidden Transitions).

Ahrens, Feenberg and Primakoff objected to the Petschek-Marshak analysis on the following grounds. The P interaction contributes through the matrix element $\int\beta\gamma_5$ connecting the nuclear states. When this matrix element is evaluated on the basis of reasonable nuclear potentials, then its magnitude turns out to be about a factor 1000 smaller than the size needed by Petschek and Marshak for their explanation of RaE.

Brysk, in his discussion, presented another evaluation which gave $\int\beta\gamma_5$ more nearly the magnitude needed. However, as did Feenberg, et. al., he must make drastically simplifying assumptions. Instead of relating $\int\beta\gamma_5$ directly to nuclear energies, he uses one-particle relativistic wave functions of a character designated by the shell model. He believes that at least the phase of $\int\beta\gamma_5$ should be given correctly by such a procedure. For this he finds just the destructive interference needed in the Marshak-Petschek analysis, when he adopts the interaction phases indicated by T + P in the conventional formulations. A similar procedure applied to the Tl^{206} case gives constructive interference instead. This fits in with the shorter comparative half-life of Tl^{206} ($10^{2.5}$ shorter than RaE, See Classification of Once-Forbidden Transitions). The contrast between the RaE and Tl^{206} cases is thus to be understood as following from the fact that RaE has one neutron and one proton outside closed shells, while Tl^{206} has one nucleon missing from each of the same closed sets of shells. Such a consideration helps remove some of the "ad hoc" character from the Marshak-Petschek conclusions about RaE.

Konopinski also discussed Ruderman's evaluation of $\int\beta\gamma_5$, which has a perhaps even less certain basis but suggests wider ramifications. It is easy to see that if the neutron were treated as a Dirac anti-particle (negative energy wave functions $\sim \gamma_5 \times$ positive energy functions) then $\int\beta\gamma_5$ would have the large order of magnitude characteristic of $\int\beta$ (the S interaction moment). This would settle nothing since it would merely interchange the role of the S and P interactions in β-decay. However, Ruderman shows that it may be a natural consequence of the meson theory of nuclear forces that the positive and negative energy nucleon states be strongly coupled in nuclei. This is to be expected from a γ_5 coupling of each nucleon to the meson fields of other nucleons. Ruderman estimated a resulting magnitude for $\int\beta\gamma_5$ in RaE, roughly consistent with the Marshak-Petschek requirement. It remains to be seen whether such evaluations of the P interaction can account for the variation with A which Nordheim's study seems to indicate (see Classification of Once-Forbidden Transitions).

Ruderman's evaluation can be questioned. He applies first order perturbation methods to find the effect of the nuclear meson field on an individual nucleon. Brueckner, Watson and others have shown that the next higher order perturbation may almost wholly suppress such effects as Ruderman makes use of. There is some indication that each successive order of perturbation alternately suppresses and enhances the effects in question.

Brysk asserted that Ruderman's evaluation cannot account for the contrast between the RaE and Tl^{206} cases, being a method, like that of Feenberg, et al., above, which fails to treat the individual properties of nuclei in a given region of the periodic table.

One can at present perhaps conclude only that the case of RaE offers crude experimental evidence for sizable effects due to the P interaction. No other explanation for this singular case seems accessible at present. Goldhaber mentioned the possibility advanced some time ago that there exists a close RaE isomer which complicates the spectrum. However, the initially uncertain evidence for this seems to have evaporated.

The Twice-Forbidden Transitions

Brysk indicated what the shell model could clarify concerning an unusually detailed type of information available relative to twice-forbidden β-decays with $\Delta I = 2$. This is the ratio of two matrix elements, conventionally denoted as A_{ij} and T_{ij}, for which values are determined in the fitting of the twice-forbidden spectrum shapes.

The shapes in question deviate from the statistical shape by a factor of the form $p^2 + \lambda q^2$, where p and q are electron and neutrino momenta. λ is a parameter depending on the magnitude and sign of A_{ij}/T_{ij}. For the measured spectra:

Cases	Transition	(Brysk)	(P-D)
$Cs^{135,137}, I^{129}$	$g_{7/2} \rightarrow d_{3/2}$	7	10
Tc^{99}	$g_{9/2} \rightarrow d_{5/2}$	2.4	2
Cl^{36}	$(d_{3/2}{}^2)_2 \rightarrow (d_{3/2}{}^2)_0$	1	0.6

Brysk obtains his λ values from the A_{ij}/T_{ij} ratio reported by the experimenters. The last column gives values obtained directly from published shapes by Peaslee and Davidson. The column labelled "Transition" specifies the state characters assigned from the shell model.

Brysk used one-particle state functions of the appropriate characters to evaluate A_{ij}/T_{ij}. The results depend on whether he employs a pure Tensor β-interaction or an ST combination. He was able to obtain fair agreement with the above experimental values with the pure T interaction. The agreement was somewhat less satisfactory with an S + T combination.

In general, the T interaction by itself has been adequate to account for twice-forbidden β-decay. The addition of other forms has usually added contributions which cannot easily be distinguished from terms already supplied by T. Thus, the uncertainities in the evaluation of λ, plus the gross oversimplifications needed for its theoretical evaluation, have conspired to prevent definitive conclusions.

The Universal Fermi Interaction

Konopinski commented on the various ramifications of the conclusion that the β-interaction needs an ST(P) combination for its expression.

The coincidence of the strengths of interaction in the meson decay and capture with the β-decay has led to the hypothesis of a "Universal Fermi Interaction" among all types of fermions. It then becomes interesting to see what consequences for the muon processes follow if the ST(P) form of interaction is applied to them.

The one experimental datum, presently accessible, which is sensitive to the form of interaction is the spectrum of electrons from the muon decay: muon $\rightarrow$ electron + two neutrinos. The possible spectra are most simply described by a parameter $0<\rho<1$: $\rho=0$ designates a spectrum with a vanishing intensity at the end-point energy and as ρ increases, the spectrum has an increasingly finite intensity at the end point. The various measurements up to now disagree with each other and any value of $\rho = 0$ to 0.4 seems about equally probable.

The theoretical predictions from the ST(P) form of law are subject to various ambiguities. First, its significance for the μ-decay depends on which particles in the μ-decay are taken to correspond to which particles in the β-decay. Three different orderings are possible, known as "Simple Charge Exchange", "Charge Retention" and "Antisymmetrical Charge Exchange". Second, the results depend on the precise relative magnitudes and phases of the three component interactions: ST(P). It is known only that $|G_S/G_T| = 0.55$ to 1 and that G_P is at most of the same order of magnitude as G_T. (see above).

Konopinski employs rather speculative theoretical arguments to conclude that the "Antisymmetrical Charge Exchange" is the most likely ordering to be correct. Expressed more physically this means that the two neutral particles emitted in the μ-decay are <u>like</u> neutrinos, one is not an anti-neutrino. Konopinski's argument is based on the necessity of making unformulatable various transformations among fermions which would contradict experience, although a "Universal Interaction" would seem to predict their occurence. He used a simpler modification of Yang and Tiomno's approach. Important in the argument is the prevention of theprocesses: $\mu \rightarrow e + e^+ + e^-$ and $\mu^- + P \rightarrow e^- + P$, which do not occur.

The consequences of the above considerations for the μ-decay spectrum are: $\rho = 0$ if the ST(P) law is more specifically $G_S(S+P) + G_T T$; $\rho < 0.05$ if $G_P = 0$; $\rho < 0.15$ if $|G_P| \leq |G_T|$. To get as high as $\rho = 0.4$, it would be necessary to have $G_P{}^2/G_T{}^2 \gtrsim 6$!

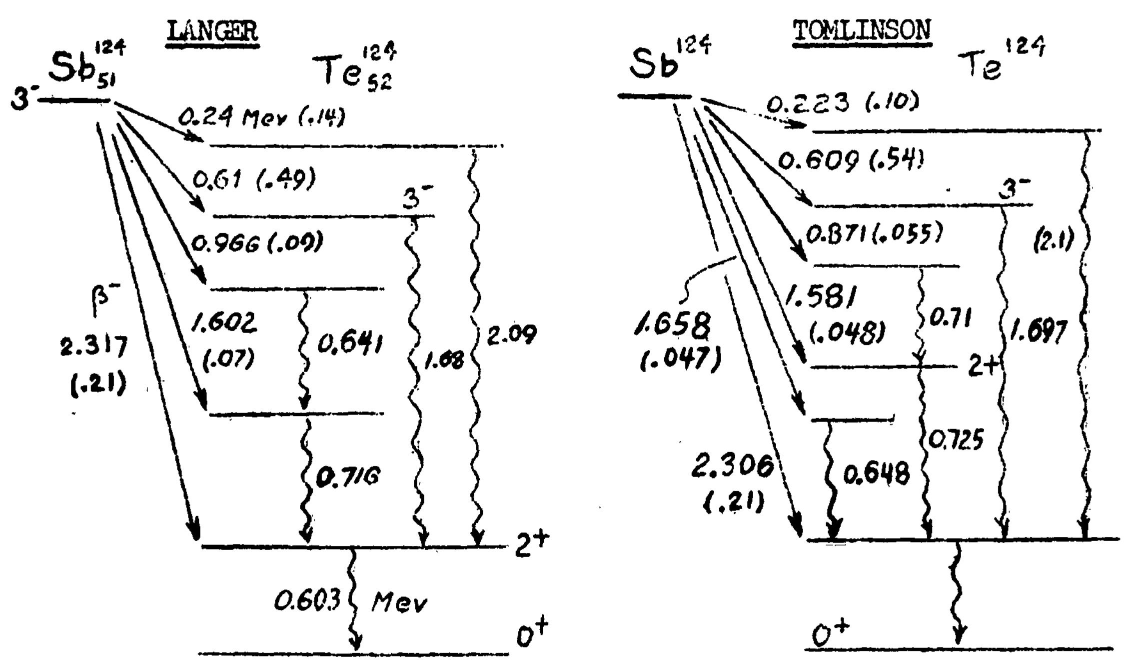
The most plausible law for a "Universal Interaction" between fermions would be S-A-P, which is antisymmetric in any pair of the fermions. Unfortunately, the evidence of β-decay is against it. (It predicts $\rho = 0.5$ for the μ-spectrum). Any equally simple theoretical criterion leading to an ST(P) law is difficult to find. It may be noted that, in a non-relativistic limit: $S+T+P \sim 1 + \underline{\sigma}_H \cdot \underline{\sigma}_L$, i.e. the Bartlett spin exchange.

The apparent complexity of the ST(P) combination might indicate that this is merely a phenomenological resultant of a more fundamental interaction between primary fields. For example, it can be asked whether a "bare nucleon" within the cloud is the essential entity. On this basis, perhaps only the μ-decay interaction, in which no nucleon participates, should be expected to show simple properties. Further, there are now two ways conceived in which the β-interaction may be a resultant of deeper interactions. A way originally introduced by Yukawa is to regard β-decay as a two step process in which a "β-meson" is first emitted by a nucleon, then the β-meson decays into electron and neutrino. Attempts along this line have not been promising so far. The work of Ruderman calls attention to another way. He showed that the pseudoscalar interaction may take on some of the features of a scalar interaction in a complex nucleus. Thus the effective form of the interaction may vary from nucleus to nucleus and the conclusions about the form must be made on the basis of properly restricted data.

The Case of Sb^{124}

Almost all once-forbidden, $|\Delta I| \leq 1$, spectra are reported to have the statistical shape. This can be understood theoretically if the Coulomb energy at the nucleus is sufficiently greater than the electron's maximum kinetic energy (see The Phenomenological Deviation, Argument V). However, deviations from the statistical shape should become observable when the kinetic energy is still somewhat less than the Coulomb energy. The theoretical explanation of the statistical shape will not be satisfactorily established until cases of such deviation are found and measured. The most interesting aspect of the Sb^{124} problem is the possibility that its highest energy β-spectrum is of the type in question. However, the Sb^{124} decay scheme is so complex that its clarification is essential before conclusions can be drawn. Langer, Tomlinson and Metzger all discussed the various complexities involved.

The decay schemes tentatively proposed by Langer and by Tomlinson are:



The highest energy β-spectrum deviates from the statistical shape. In earlier work, it was fitted with the theoretical shape characteristic of the "unique" ($\Delta I = 2$) spectra. However, $\beta\gamma$ correlations in the parent Sb^{124} indicated a 3- or, less probably, 4^+ ground state. On the other hand, Metzger established the indicated cascades $3^- \rightarrow 2^+ \rightarrow 0^+$ in Te^{124}, by correlations between the 0.6 and 1.7 Mev γ-rays (see, moreover, Even-Even Nuclei for arguments that the first excited state is 2^+). Thus, a "unique" shape seems out of the question: the spin assignments indicate a once-forbidden ($3^- \rightarrow 2^+$) shape or, with smaller probability a twice forbidden ($4^+ \rightarrow 2^+$) spectrum. log ft = 10.3, about midway between the usual once-forbidden and twice-forbidden values. Nordheim pointed out that a lower ft than usual for the second forbidden transition with the high energy released here may well be expected.

Langer discussed evidence in favor of the $\Delta I = 1$, once-forbidden interpretation of the highest energy β-spectrum. His chief argument is that the lower energy spectra, obtained after subtraction of the highest energy spectrum, then exhibit end-points in satisfactory conformity with the γ-ray energies. The most unsatisfactory feature in the earlier fittings of the "unique" shape had been that the lower end-point energies disagreed with the γ-ray energies uncomfortably far outside experimental errors. However, the decay scheme of the γ-rays is still in dispute as will be seen below.

Tomlinson's measured spectrum disagrees with Langer's by a slight amount, but sufficiently for the difference to be critical. He finds closer agreement with the unique shape. He further finds that the predictions of the theory for a twice-forbidden spectrum are sufficiently elastic to fit his observations also. Thus Tomlinson would assign 4^+ rather than 3^- to the Sb^{124} ground state.

Both Langer and Tomlinson have evidence for 0.60, 0.72, 0.64 and 1.68 Mev γ-rays. Langer's group supplemented the internal conversion measurements with photoelectron and scintillation spectrometer measurements which also detected the 2.1 Mev γ-ray. Tomlinson includes another, 0.71 Mev γ-ray in his scheme on the basis of evidence offered by Metzger (below). Such a γ-ray would be superposed on the 0.72 Mev γ-ray in Langer's photoelectron measurements.

Metzger discussed his $\gamma\gamma$ and $\beta\gamma$ coincidence measurements on Sb^{124}. In both types of measurement, he set one channel on the high energy side of the 0.72 Mev peak; he believes that in this way he avoided interference from the 6 to 10 times as intense 0.60 Mev peak, and also from the half as intense 0.64 Mev peak. The second channel revealed a 0.72 ± 0.03 Mev γ-ray in coincidence with the 0.72 Mev γ-ray in the first channel. Its intensity was less than 1/5th of the first 0.72 Mev γ-ray's intensity. No peak was found at 0.64 Mev, as would be expected on the basis of Langer's scheme. Hence Tomlinson's scheme, which makes room for the second 0.7 Mev γ-ray, seems to be favored. However, Tomlinson's analysis of his β-spectrum seems to require more than twice as intense a second 0.7 Mev γ-ray as Metzger found.

The interpretation of the $\beta\gamma$ coincidences is made uncertain by the fact that it requires detection of a deviation in a Fermi plot based on pulse height measurements of electron energies. Metzger detected no deviation as he swept through electron energies from 1.6 Mev down to less than 0.5 Mev. He thus would give the β-rays feeding the second 0.7 Mev γ-ray a small intensity, in conformity with the intensity he finds for the γ-ray. Such a low energy β-intensity is considerably less than either Tomlinson's or Langer's β-spectrum analyses would lead one to expect.

Deutsch pointed out that there is a considerable discrepancy in the energies assigned by the two decay schemes to the middle β-ray group (0.966 vs. 0.871 Mev). $\beta\gamma$ coincidence spectrum measurements should be able to settle this point.

The Spectrum of C^{14}

Mize reported a disquieting result of a measurement of the C^{14} spectrum. He and Zaffarano used a proportional counter technique which they tested extensively and successfuly on a series of well known spectra. For C^{14}, they obtained a Fermi plot which had the linear behavior reported by others, except that the intensity dropped quite sharply for electron energies below about 50 Kev. Yet their techniques continued to yield the expected (linear Fermi plot) behavior down to 10 Kev for S^{35} and Pm^{147}. The sources in all cases were of comparable thickness, ~10 g/cm^2, unusually thin because a large solid angle is available for detection.

The low energy drop has been reported for other cases, notably for Ru^{103}, by Kondaiah. No completely convincing explanation can be readily offered.

Wu has recently reported the linear Fermi plot for C^{14} down to ~25 Kev., but with a thicker source. Earlier work by Langer and Warshaw also showed the linear behavior. This is not complete proof because source thickness usually tends to hide such effects as reported by Mize and Zaffarano. Langer used various source thicknesses; he found that a drop could occur with too thick a source, and he eliminated by using a thinner one. His trials were however limited to thicker sources than that of the present experiment.

K-Capture in Zn^{65}

Haynes reported a thorough investigation of the orbital electron capture in Zn^{65}. He detected K-Auger electrons and internal conversion electrons whose intensity could be directly compared to that of the positrons. 45% of the transitions consist of pure orbital capture leading to an excited Cn^{65} state which radiates a 1.11 Mev γ-ray. Haynes finds that 1.97 ± 0.23% of the transitions consist of 325 Kev, allowed, positron decay to the ground state. The ratio of the K-capture leading to the ground state, to the positrons, is 28.0 ± 3.2. This is in very good agreement with theoretical expectations.

THE γ-RADIATION

The γ-Lifetime, Energy Relation

Unlike β-emission, the γ-radiation is expected to follow the long established fundamental laws of electromagnetic radiation. For nuclei, these are embodied in formulas such as those of Weisskopf. Formulas of this type give the γ-radiation probability according to its multipole character, its energy, and the character of the nuclear states connected in the transition. The contribution of the last factor is proportional to the square of an appropriate nuclear matrix element, $|M|^2$. It is specific only when special assumptions are made concerning the nuclear states involved. In practice, one adopts state characters indicated by the shell model for the last one or two odd nucleons. This procedure establishes what Breit referred to as the "Weisskopf unit" for $|M|^2$. It is convenient to compare measured $|M|^2$ values, i.e. calculated from the above formulas and the observed γ-radiation probability, with the "Weisskopf unit".

As is to be expected, with most nuclei having many particles outside closed shells, the γ-radiation lifetimes on the whole deviate greatly from the formulas based on single particle transitions. Goldhaber pointed out that one or two cases of agreement with the single particle predictions exist and these are for one particle outside closed shells. He stressed the importance of seeking out for investigation all the cases in which a single particle transition might be presumed.

The difference between single neutron and proton transitions should then also be taken more seriously. Moszkowski and others have developed formulas which make the theoretical distinction.

E 1 Transitions

Sunyar summarized evidence on the probability of electric dipole radiation.

For $A \lesssim 17$, the radiation widths in p,γ reactions indicate a remarkable uniformity of $(2I_f + 1)\ |M|^2 \approx 0.2$. By $|M|^2$ here is meant the quantity measured in "Weisskopf units". Thus a somewhat smaller probability is indicated than would be predicted for single-particle transitions.

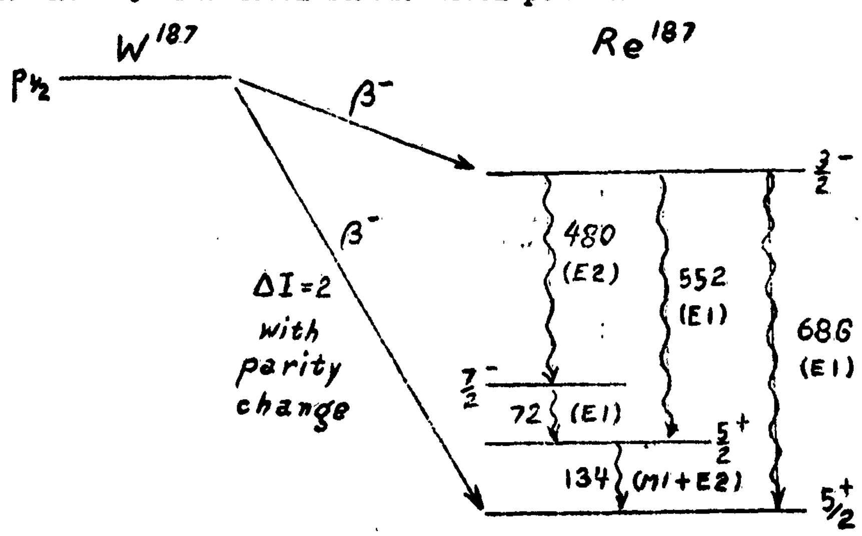
For $A \approx 50$, the neutron capture γ-radiation widths correspond to probabilities $\sim 1\%$ of those expected from the single particle model. Still, the E 1 widths are ~ 100 times as large as the E 2 or M 1 radiative widths.

Sunyar was chiefly concerned with data on γ-rays following radioactivity, which can be summarized in the table:

Nucleus	Energy (Kev)	$\|M\|^2$
A^{38}	1600	$\sim 2(10)^{-7}$
Sr^{88}	910	$\sim 10^{-4}$
Te^{124}	1700	$\sim 3.5(10)^{-5}$
Hf^{177}	206	$\sim 10^{-5}$
Hf^{177}	318	$\sim 10^{-8}$
Re^{187}	72	$\sim 2(10)^{-7}$
Re^{187}	552	$\sim 2(10)^{-8}$
Re^{187}	686	$\sim 5(10)^{-7}$
Np^{237}	72	$\sim 3(10)^{-6}$

The last column gives $|M|^2$ in "Weisskopf units" as deduced from observed γ-half-lives. One sees that the single-particle transition model very greatly overestimates the E 1 radiation to be expected in heavy nuclei.

The intricate investigation needed to identify the E 1 transitions of the table was discussed in detail for an example of Sunyar's own work: Re^{187}. The level scheme developed was:



Delayed coincidences between the 480 Kev and 134 Kev radiation show that the 72 Kev radiation has a $5(10)^{-7}$ sec half-life and is preceded by the 480 Kev γ-ray. Conversion X-rays due to the latter indicate that it is E 2. The K conversion of the 72 Kev γ-ray is compatible either with E 1 or E 2 but the lack of L conversion definitely indicates E 1. Finally, the relative conversion of the 480, 552 and 686 Kev γ's indicates that the last two are also E 1.

Classification by Multipolarity

It is preferable to have independent evidence as to the multipolarity of the γ-radiation; this information is obtainable from observations on the γ-rays after their emission, whereas the radiation intensity depends on the nuclear states. One of the observable effects most sensitive to multipolarity is the internal conversion. The interpretation of internal conversion coefficients rests on a secure type of theory, but reliable theoretical tables are as yet only partially completed under the leadership of Rose. Meanwhile, a semiempirical approach to the interpretation of K/L internal conversion ratios has been developed by Goldhaber and Sunyar.

Goldhaber discussed recent revisions in the empirical curves of the K/L ratio vs. Z^2/E ($E = \gamma$-energy), for each multipole. The M 4 curve is now substantially lower than the original one at low values of Z^2/E, largely due to work by Graves, Langer and Moffat. The M 1 curve is somewhat lower for high Z^2/E becuase of diverse new findings. The M 2 curve is now better established through the elimination of misinterpreted cases.

Mihelich discussed the use of $L_{I,II,III}$ conversion ratios as a sensitive method of distinguishing multipolarity, especially when, most usually in heavy elements, the γ-energy is sufficient for K conversion.

The available theoretical L-conversion ratios are fragmentary; they are limited to non-relativistic calculations, uncorrected for screening. However, insofar as checking has been possible, the empirical conversion ratios agree well with the theory. The L_{III}/L_I ratio for M 4 radiation is particularly well-checked.

A striking example of the usefulness of the method is afforded in the comparison of ~130 Kev γ-rays in Hg and Au. The L_{II}/L_{III} ratio is ~1.1 in Hg, ~ 2.5 in Au, as expected for E 2 and E 3 radiation, respectively.

Also useful is the contrast shown in the conversion of E 2 and M 1 radiations. The E 2 radiation in heavy elements is strongly converted in $L_{II,III}$, very little in L_I (1/30th for a 77 Kev Au γ-ray).

On the other hand M 1 radiation yields a high $L_I/L_{II,III}$ ratio (~ 20 to 30 for a 77 Kev Au γ-ray). This makes the method particularly sensitive to mixtures of E 2 and M 1 (see below).

Mixed Multipoles

The intensity of radiation by a classical charge distribution has always been expected to be about the same for the ML as for the EL + 1 multipole. Mixtures of such radiations may then be expected, since both need the same parity change; $(-)^{L+1}$.

Goldhaber and Sunyar found empirically that the ML γ-radiation is relatively much stronger, of the same order as EL. When the latest nuclear radiation formulas were developed this became understandable. Because of the intrinsic magnetic moments, for example, the classical relationship between the electric and magnetic multipoles is lost. If ML and EL transitions always radiated equally strongly, mixtures would not be expected, since these occur for opposite parity changes.

Goldhaber discussed a new development: the finding of ML, EL-1 mixtures (see Steffen's discussion below). He suggested that this is correlated with another set of observed facts.

The ML γ-transitions are on the whole found to be much slower than the single-particle radiation formulas predict. However, their variation from core to core is surprisingly smooth and closely parallels the variation predicted. On the other hand, the EL radiations vary irregularly in lifetime, by large factors from core to core. On this basis, it is not surprising that cases occur in which EL-1 transitions radiate comparably with ML, thus giving an observable mixing of the two types.

Steffen discussed well-investigated examples of both types of mixing: ML, EL $\pm$ 1. He uses the very sensitive method of directional correlations between successive γ-emissions. The correlation function has the form:

$$W(\vartheta) = \sum_{k=0}^{m} A_{2k} P_{2k} (\cos \vartheta)$$

where ϑ is the angle between the γ-rays, m is the magnetic quantum number of the intermediate state, and the coefficients A_{2k} depend in a known way on the energies and the multipolarities of the γ-rays.

Caution must be observed in interpreting the experimentally measured coefficients, A'_{2k}. In the first place they are integrals over the finite solid angles of radiation actually detected; this is calculable for detectors of calibrated efficiency. In the second place, A'_{2k} may also be multiplied with an attenuation coefficient, G_{2k}(0 to 1), due to extra nuclear (atomic) effects causing disorientation of the intermediate nuclear spin direction (change of m called spin-coupling by Rose, below). The latter effect is small if the intermediate state radiates the second γ-ray rapidly enough, and can be miniimized by providing the nucleus with a suitable atomic environment. Steffen compares different sources: various liquids and, if possible, ionic crystals in which the interesting nuclei are embedded. He considers the results significant when they are the same for the different sources.

The $\gamma\gamma$ coincidence curves obtained by Steffen in several cases are represented by:

Cd^{114}: $W = 1+0.114\ (\pm .008)\ P_2\ (\cos\vartheta) + 0.012\ (\pm .006)\ P_4\ (\cos\vartheta)$

Pt^{196}: $W = 1+0.092\ (\pm .008)\ P_2\ (\cos\vartheta) + 0.314\ (\pm .010)\ P_4\ (\cos\vartheta)$

Sr^{88}: $W = 1 -0.0645\ (\pm .0015)\ P_2\ (\cos\vartheta) + 0.001\ (\pm .0015)\ P_4\ (\cos\vartheta)$

In the first two cases the first γ connected $I_1 = 2$ with $I_m = 2$, while in Sr^{88} the spin change was $3 \rightarrow 2$. The second γ-ray transition was $2^+ \rightarrow 0^+$ in all three cases: E 2 radiation.

The following Tables give the interpretation for the above cases together with similar results obtained by others.

Nuclei	γ-Energies (Mev) γ_1	γ_2 (E 2)	γ_1 % E 2 in M 1	1 - Particle Theory
Se^{76}	0.65	0.56	34-80%	1%
Cd^{114}	0.72	0.55	3%	1%
Te^{122}	0.68	0.56	80%	1%
Pt^{194}	1.48	0.33	98%	5%
Pt^{196}	0.36	0.33	95%	1%
Hg^{198}	0.68	0.41	60%	1%

One sees that the Weisskopf single-particle predictions greatly underestimate the E 2 radiation relative to M 1 in these cases. Only Cd^{114} is relatively close to agreement. All the cases have many particles outside closed shells, which suggests that deviation be expected from the single-particle formulas. Steffen emphasized that the existence of E 2 transitions fast enough to mix with M 1 points to "classical" radiation by collective motion of the large charged core (see THE NUCLEAR CORE).

Two cases of E 1 + M 2 mixture are fairly clear:

Nuclide	γ-energies (Mev) γ_1	γ_2 (E 2)	γ_1 M 2 in E 1	1-Particle Theory
Sr^{88}	0.91	1.78	0.007%	0.00015%
Te^{124}	1.7	0.60	$\lesssim$0.1%	0.0001%

Considering the finding (see preceding section) that E 1 radiation is greatly overestimated by the Weisskopf formula, the descrepancy here may be ascribed to a smaller overestimate of M 2 transition probabilities.

Mihelich's observations on L conversion (see above) have also turned up some mixed transitions: 5 cases in odd proton nuclei, four of them $d \rightarrow S$ transitions. Characteristically, comparable L_I and $L_{II,III}$ conversions are found, in contrast to only L_I conversion for pure M 1 cases, and only $L_{II,III}$ conversion for pure E 2 cases. One case of an E 3 + M 4 mixture was also found.

γ-e Directional Correlations

Rose discussed means of avoiding certain experimental difficulties in direction correlation experiments. Usually, to avoid uncertainty due to the coupling of the intermediate spin with extranuclear fields, various sources are compared — dilute solutions being important ones (see Steffen's discussion above). This is not possible if one of the particles is an electron (β or conversion), since the source must then be very thin to avoid scattering.

For γe correlation, the angular distribution given for $\gamma\gamma$ correlation above is modified to

$$W(\vartheta) = \sum b_{2k} A_{2k} G_{2k} P_{2k}(\cos\vartheta)$$

where $b_{2k}A_{2k}$ is the theoretical coefficient in the absence of the intermediate-spin coupling. With the coupling, the attenuation G_{2k} is naturally the same as for $\gamma\gamma$ correlation from the same source. Hence b_{2k} can be found by performing both experiments:

$$b_{2k} = (b_{2k} A_{2k})'_{\gamma e} \big/ (A'_{2k})_{\gamma\gamma}$$

where the primes indicate measured coefficients.

An alternative procedure is to measure the γe correlations seperately for K and L conversion electrons. Then the ratio of the experimental coefficients is b (K)/b(L) regardless of the attenuation. Unfortunately, relativistic theoretical values for b(K) and b(L) are not yet available, and in the non-relativistic limit, b(K)/b(L) = 1 for pure EL radiations.

$\gamma\gamma$ Polarization-Direction Correlations

Extremely sensitive results can be obtained by measuring the correlation between the direction of one γ-ray and the polarization of the other. The problem was first treated by Hamilton. The correlation function has the form $W = \sum A_\nu \mathcal{P}_\nu(\vartheta, \varphi)$ where $\mathcal{P}_\nu$ is no longer simply a Legendre polynomial as for the direction correlations. It is a function of the angle ϑ between the directions of the two γ-rays, and of another angle φ between the plane of polarization of one γ-ray and the plane containing the directions of both:

$$\mathcal{P}_\nu = P_\nu(\cos\vartheta) + (-)^{\sigma_1}\alpha_\nu(L_1)\cos 2\varphi \, P_\nu^2(\cos\vartheta)$$

if the γ-ray whose polarization is being measured is EL_1 or ML_1; $\sigma_1 = 0$ for EL_1, $\sigma_1 = 1$ for ML_1. $\alpha_0(L) = 0$ and

$$\alpha_\nu(L) = \frac{(\nu-2)!}{(\nu+2)!}\,\frac{2\nu(\nu+1)L(L+1)}{\nu(\nu+1)-2L(L+1)}, \quad \nu \geq 2$$

The term with this coefficient vanishes for purely directional correlations ($\cos 2\varphi = 0$).

Rose discussed the analysis of results obtained when coincidences are measured between an ordinary γ-detector in one direction and a polarization-sensitive detector in a direction making an angle ϑ with the first.

Rose first presented the general conditions under which no polarization anisotropy would appear: for EL, ML or ML, EL cascades; also, for E 1 E 2, M 1 M 2, E 2 E 1 or M 2 M 1 cascades. This is presuming that the efficiency of detection is the same whether one γ-ray or the other is the one registered by the polarization sensitive detector.

In the cases just mentioned, one of the transitions involves a change in parity i.e. there is an "overall" parity change. Rose was able to exhibit a series of simple relations between $\gamma\gamma$ directional correlations and direction-polarization correlations for cases of no overall parity change (er. E 1 M 2).

Further, Rose presented an extensive analysis of the measurements when the efficiency of polarization detection is different for one γ-ray and the other. The results are important because they enable one to identify the individual parity changes in the two transitions being correlated. Finally, Rose developed the correlation function for cases in which one of the γ-rays is radiated in a mixed transition (see Mixed Multipoles, above). The function $\mathcal{P}_\nu$ described above is now replaced by another of the same general form with new coefficients replacing α_ν. The correlation function W now also depends on the ratio δ of the components in the mixture. The appearance of δ prevents anything but accidental cancellations of the anisotropy.

ENERGY VS. CHARACTER OF EXCITED STATES

Odd A Nuclei

Correlations between energy levels and their state characters are to be expected. The energy and character of many first excited states has been determined from the study of isomers, made convenient by their long lives in the states. Goldhaber, in his discussion, exhibited a plot of the odd mass numbers, A, of isomers against the odd nucleon numbers (neutron or proton). This showed that "islands of isomerism" occur almost exclusively during the filling of the last shells preceding magic numbers. There are one or two exceptions to this, of a kind discussed below (Mo^{93}, see after Even-Even Nuclei). Only odd neutron isomers occur in the 50-82 and 82-126 regions.

At the closing of the N or Z = 50 shells, there is competition between $p_{1/2}$ and $g_{9/2}$ orbitals. This gives an opportunity for studying the $E(p_{1/2}) - E(g_{9/2}) \lesssim 0$ energy difference. It is found to vary smoothly as pairs of neutrons are added to the core underlying the $p_{1/2}$ and $g_{9/2}$ states.

The best data are provided by three series of odd proton isotopes: $Y_{39}{}^{87,89,91}$, $Nb_{41}{}^{89,91,93,95,97}$, $Tc_{43}{}^{93,95,97,99}$. The $p_{1/2}$ energy has a minimum relative to $g_{9/2}$ exactly at neutron number N = 50 in each of the isotopic series. Goldhaber points out that the tighter core of N = 50 should be expected to have a relatively larger interaction with a $p_{1/2}$ proton than with the more spread out $g_{9/2}$ orbital. The energy $E(p_{1/2})-E(g_{9/2})$ also increases as Z increases towards 50. Apparently, the more numerous $g_{9/2}$ protons tend to stabilize each other in that orbit more than the two $p_{1/2}$ protons can help each other.

The last phenomenon is also exhibited in fragmentary series of odd neutron isotopes. The data on these is less abundant because the odd neutron isomers are divided into a second kind. The odd neutron groups $g_{9/2}{}^{3,5,7}$ most often produce an even I = 7/2 state, instead of the resultant $g_{9/2}$. The energy difference $E(7/2+) - E(p_{1/2})$ also shows a smooth behavior with the addition of neutron pairs. The isotopic series $Se_{34}{}^{77,79,81}$ and $Kr_{36}{}^{79,81,83}$ provide the data. Now there is an energy minimum at the exact middle of the $g_{9/2}$ shell, N = 45, where the 7/2+ state gains the greatest stability relative to $p_{1/2}$.

Regularities also appear in the "islands" preceding N = 82 and 126. No comparable evidence turns up in the comparison of isotones, when it is the proton pair number which varies, as was brought out by a comment from Mitchell.

Mihelich discussed the regularities in the low excited states of a series of gold isotopes. These are daughters of mercury isotopes produced from the Au^{197} (p,xn) Hg reactions. Proton energies up to 105 Mev eject up to 9 neutrons. Data could be obtained on the γ-radiations of all the gold isotopes with N = 112 to 118. The radiating states could be characterized after determining the γ-ray multipolarities by means of L conversion ratios (see Classification by Multipolarity).

The ground states of the even N Au isotopes were all presumed to be $d_{3/2}$, in agreement with the well-established assignment for stable Au^{197}. Excited $s_{1/2}$ and $d_{5/2}$ states are identified in each isotope.

Mihelich reported that $E(d_{5/2})-E(d_{3/2})$ was nearly constant in the series (250-280 Kev). Axel commented that this is not surprising if only a spinless core varies from member to member in the series. More variation should be expected for $E(s_{1/2})-E(d_{5/2})$ since the S-orbit overlaps the core more than does a d-orbit. Indeed, the $s_{1/2}$ energy rises relative to $d_{3/2}$ from $\sim$40 to 77 Kev., as neutron pairs are added. Mihelich compared this with E $(d_{3/2})$ - $E(s_{1/2})$ as found in the odd neutron series Sn_{50}, Te_{52}, Xe_{54}. The last energy difference decreases with the addition of protons, and thus again $E(S_{1/2})$ rises relative to $E(d_{3/2})$.

Even-Even Nuclei (Assignments)

Scharff-Goldhaber reviewed current knowledge concerning the first two excited states of even-even nuclei. Almost all the first excited states are 2+ (even I = 2). More than a third of the second excited states are also 2+ , about an equal number being 4+ instead. (see Mixed Multipoles for a discussion of the γ-radiation during $2^+ \rightarrow 2^+ \rightarrow 0$ transitions). The results in general point to a working hypothesis that the nth excited state will have $I \leq 2n$, and two consecutive ones $\Delta I \leq 2$.

Two alternative theories of the excited state characters have been advanced. One of these presumes that the low excited states all arise from a single nucleonic configuration. The configuration is taken to be j^n for n like nucleons in the last unfilled j-orbital. If now zero-range forces are assumed, a level order can be calculated: I = 0, 2, 4 . . . 2j-1, all of even parity, with the ground state put first. Flowers has shown that the order: I = 0, 2, 2 . . . is also obtainable in some cases; this was shown specifically for four j = 7/2 nucleons interacting with forces of sufficiently long range.

Ford discussed the alternative "collective model" of the nuclear excitations. The first excited state now represents a rotation of the nucleonic core, underlying the unfilled orbitals. Consideration

Scharff-Goldhaber pointed out that an odd A nucleus formed by the addition of a single nucleon to an even-even nucleus generally has a lower first excitation energy than the even-even nucleus. This is to be understood as the excitation of the single nucleon superposed on the even-even core which is still unexcited in the first odd A state.

Goldhaber discussed a striking example of an odd A nucleus which behaves like an even-even nucleus in its first three excited states. This is Mo^{93m}, an isomer with $N = 51$, in striking exception to the rule that isomers do not occur just after a magic number shell ($N = 50$) is filled. The half-life of the third excited state is 6.75 hr in spite of an excitation of 2.428 Mev. This implies a very high spin, especially since no direct radiation to the ground state occurs. There is instead a cascade, starting with 263 Kev E 4 radiation, followed by what are probably two successive E2 radiations. Successive core excitations to $I = 2$, 4 and 8 are thus indicated. Meanwhile the odd $g_{7/2}$ neutron may well remain unchanged in the successive states.

of the core motion alone is not sufficient, however, for the explanation of the marked shell effects observed in the excitation energy (see below). Thus, a strong coupling of the core to the extra-core nucleon is assumed. There follows a "collective" motion and the second and higher excited states may involve excitations of the outer nucleons superposed on the core motion. The level orders: $I = 0, 2, 4$ are obtained for most cases; $I = 0,2,0$ was obtained for two extra-core nucleons when $A = 100$.

The two principal exceptions to the rule that the first excited state has a 2 character are O^{16} and Ge^{72}, which have 0 instead. Scharff-Goldhaber pointed out that the pure shell model would lead to an expectation that the 2+ rule be violated when the nucleus has only filled orbitals, and particularly so if it is magic number shells which are filled. The neutrons of Ge^{72} just fill a $p_{1/2}$ orbital whereas the protons fill a $p_{3/2}$ orbital. Both the neutrons and protons of O^{16} fill the shells at magic number 8. On the other hand, Pb^{208} is also "doubly magic", yet has a 2+ first excited state. The core excitation seems to be indicated here. Determination of the first excited state character in Ca^{40} is still lacking.

Richards announced results which removed another apparent exception to the rule that the first excited state is 2+ . Earlier experiments on the inelastic scattering of deuterons on Ne^{20} were interpreted as indicating an odd parity for the first excited state of Ne^{20}. The interpretation was based on an extension of the stripping theory (see STRIPPING REACTIONS) to inelastic scattering by deuterons. Interpretations are fundamentally more straightforward when a single nucleon is used for the excitation. Richards reported that where protons are inelastically scattered on Ne^{20}, the angular distribution unambiguously requires a 2+ assignment to the excited state.

Even-Even Nuclei (Energies)

Scharff-Goldhaber displayed a plot of the first excitation energies of even-even nuclei, as a function of neutron number. There are striking peaks in the energies at the doubly magic numbers, and also at $N = 28$, 50 and 82. Somewhat less high maxima of energy occur at $Z = 28$, 50. These phenomena emphasize the importance of the shell structure in the excitations.

Equally striking are the low "valleys" of excitation energy between the peaks, especially between $N = 82$ and 126 and for $N > 126$. Ford emphasized the correlation between the occurence of the rare earth "valley" ($N = 82$-126) with the large quadrupole moments found for the nuclei here. Large deformability of the core, as indicated by the quadrupole moments, should lead to low core excitation energies. However, a considerably larger deformability than is indicated by the measured quadrupole moments seems to be required to get the very low energies observed. The agreement is still much better than is easily obtainable with the shell model alone. A somewhat weaker coupling may be needed between the core and the extra nucleons than presumed.

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NUCLEAR BOMBARDMENT EXPERIMENTS

Resonance Scattering of Protons

Richards described experiments on the elastic scattering of protons by nuclei. Over sufficient ranges of energy, resonances in the differential scattering cross-section are found. This can be most informative, as discussed below, when the resolution is sufficient for the resonance widths to be unobscured by instrumental broadening. To provide the resolution, the accurate energy control possible with van de Graaf accelerator is advantageous. Further, electrostatic beam analysis was used and a solid window between beam and scattering gas was avoided. The last step was possible with fast differential pumping between the target gas chamber and the vacuum system guiding the beam.

Analysis of the resonance scattering angular distributions allows determination of the spins and parities of the resonance levels. The most easily analyzable results are obtained for I = 0 target nuclei: C^{12}, O^{16}, Ne^{20}, Mg^{24}. Relatively low levels can be reached because the proton is lightly bound to these nuclei. Consistency with shell model expectations was found: the interpretation here is particularly simple since the resonances states are formed from the single bombarding nucleon on an even-even core.

The measured resonance widths give further information, after allowances for Coulomb barrier penetration are made. The resultant "reduced-width", γ, has an essentially simple significance. If γ is large, $\sim \hbar^2/MR$, then the single-particle description of the state is valid. If it is much smaller than this, more complicated excitations are needed to form the state. Both types of widths are found, indicating which of the various levels are formed by which type of excitation.

Stripping Reactions

Hough described experiments especially designed to reveal mixing of ℓ-values in the shell model states of nuclei. The angular distributions from (d,p) reactions on P^{31}, Cl^{35} and Sc^{45} targets were measured. When analyzed according to the stripping theory, such measurements reveal the ℓ-value of the neutron in the last oribtal of the resultant nuclei.

The Michigan cyclotron provided the deuteron beam, with an energy spread of ~ 150 Kev, at 7.8 Mev. The targets are mounted on foil wheels in a scattering chamber which was slightly tipped to enable detection at 0^{o}. Proportional counter detectors were used and coincidences required, in order to minimize background.

The stripping theory predicts the angular distribution

$$\frac{d\sigma}{d\Omega} \sim \sum_{\ell} \frac{|f_\ell|^2}{g_\ell^2} |F_\ell(\vartheta)|^2$$

with a summation over all ℓ-values permitted by angular momentum and parity conservation. F_ℓ is a known function of the scattering angle ϑ. It leads to peaks in the angular distribution: a peak at 0° for $\ell=0$, at progressively larger angles for $\ell=1,2,\ldots$. g_ℓ^{-2} is a known weight factor which suppresses higher ℓ-values in favor of the lowest possible. $|f_\ell|^2$ is the (unknown) probability of finding the neutron per unit radial distance on the surface of the core (≡ target nucleus).

The shell model predicts unique ℓ-values in simple cases. Even small admixtures of smaller ℓ-values will show up strongly because of the weighting g_ℓ^{-2}. One has here a sensitive measure of deviations from pure shell model predictions.

The P^{31} ground state is $\frac{1}{2}+$, hence an added neutron must have $\ell=0$ or 2 to produce the P^{32} ground state, 1^+ . The shell model expectation is that the neutron will enter a $d_{3/2}$ orbit. Thus, a proton angular distribution corresponding to $\ell=2$ only is predicted. Actually, an excess of protons in the forward directions is found to be superposed on the characteristic $\ell=2$ distribution. The forward excess is, in this case, differently shaped than one expects from an $\ell=0$ admixture, but it was so interpreted. From the intensities, $|f_0|^2 / |f_2|^2 \approx 3$ to 4 %.

For Cl^{35}, the analysis is much the same except that this time the forward peak has the expected $\ell=0$ shape. The admixture $|f_0|^2 / |f_2|^2 \approx 3$ to 3½% is found.

Sc^{45} has 7/2 − in its ground state and $\ell=1$, 3 or 5 neutrons can form the Sc^{46} (4+) state. An $\ell=3$ peak is expected since it is an $f_{7/2}$ orbital which is to be filled by the neutron. Actually, an $\ell=1$ admixture is found, with $|f_1|^2 / |f_3|^2 \approx 13\%$.

The observed angular distribution peaks are superposed on a uniform distribution which must first be subtracted. It amounts to more than 30% of the peak value, at each point of the distribution. It is attributed to compound nucleus scattering; i.e. some of the protons are emitted after the deuteron and the target nucleus form an intermediate compound system. Breit pointed out a serious danger in the procedure. A 30% scattered intensity corresponds to a scattering amplitude almost half as large as the peak scattering amplitude. Interference between the two types of scattering could produce effects another factor 2 as great. Hough replied that one may hope that the two types of scattering are incoherent, since the compound nucleus formation implies loss of coherence. The peaks are as distinct as is expected from the stripping theory alone.

THE NUCLEAR CORE

The Liquid Drop

The liquid drop model for the nucleus has long provided a useful means for treating various phenomena such as fission, intermediate states in nuclear reactions, and quadrupole moments. Brolley discussed new experiments on fission which exhibited strikingly that heavy nuclei indeed behave in a way expected of a liquid drop. The fission was induced by fast (14 Mev) neutrons and the angular distribution of the fragments studied from several target nuclei: Th^{232}, U^{233}, U^{235}, U^{238} and Np^{237}. In each case a marked preponderance of fragments along the direction of the neutron beam was found: 25% more than in the lateral direction. The picture is clear: the fast neutrons induce deformations along the line of impact, which develop to the point of fission. As expected, the angular distribution is iso tropic in fissions induced by thermal neutrons. The fast neutron effect contrasts understandably with the fragmentation by photo-fission: this is preponderantly lateral, in the direction of dipoles one should expect to be induced.

The Coupling to Extra-Core Nucleons

The liquid drop model is obviously inadequate for treating the marked shell effects shown in many nuclear phenomena (see, eg. Even-Even Nuclei). At least the nucleons in unfilled shells must be attributed a quasi-independent motion. The collective motion which ensues from a strong coupling between the extra nucleons and the deformable core was discussed by Ford. Surface motion of the core is split into a rotation of the deformed nucleus and vibration about the deformed equilibrium. Since the deformation is viewed as subject to the motions of the extra nucleons, one expects deformation phenomena to show themselves most , for nuclei far from magic number configurations. This has already been seen in connection with the excitation of even-even nuclei (see the statements about the rare-earth "valley" in Even-Even Nuclei).

A further consequence is enhancement of the E 2 radiation rate in even-even nuclei. The observed rates, in nuclei far from magic numbers, are distinctly larger than one expects from the two nucleons which would be responsible on the basis of a pure shell model. The deformations required to account for the observed rates are reasonably consistent with those found in other ways.

Magnetic Moments

Ford went on to discuss the magnetic moments predicted by the collective model. The latter predicts not only an addition of moments to be expected from deformations of the core, but also effects from the mixing of nucleon states due to interaction with the core motion. Magnetic moments arising from a weak coupling of the core with the extra nucleons have been calculated previously.

A brief summary of results is: a) For nuclei with one extra nucleon with $j = \ell + \frac{1}{2}$, the calculated moment falls well within the Schmidt limit, farthest in the strong coupling treatment. b) For three $j = \ell + \frac{1}{2}$ nucleons, the calculated moment is within the Schmidt line except for $j = 5/2$, when it is slightly outside. c) Calculated deviations from the $\ell - \frac{1}{2}$ Schmidt lines are of both signs and always small. Reasonable agreement with experiment is achieved when the anomalous nucleon moment is assumed to be reduced by 0.8 unit by the presence of nuclear matter. Most nuclei for which collective effects are expected to be negligible, fall on the new Schmidt lines, while moments of nuclei expected to show strong core deformations fall farthest off the new lines. The well-known Bi^{209} anomaly still remains.